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Major P. A. MACMAHON, D.Sc., Sc.D., LL.D., F.R.S., PRESIDENT, in the Chair.

Ronald Malcolm, Headley Grove, Headley, Epsom, was balloted for and duly elected a Fellow of the Society.

Sixty-eight presents were announced as having been received since the last Ordinary Meeting, including, amongst others:—

Specola Vaticana, Catalogo Astrografico, vol. 3, Zone + 62°, presented by the Observatory.

T. J. J. See, Electrodynamic Wave-Theory of Physical Forces: Discovery of the Cause of Magnetism, Electrodynamic Action, and Universal Gravitation.

On the Origin of Ptolemy's Catalogue of Stars. Second Paper. By J. L. E. Dreyer, Ph.D.

In a paper on this subject communicated to the Society last year (*Monthly Notices*, **77**, 528) I drew attention to the fact that the methods adopted by Ptolemy for determining the places of standard stars of necessity introduced large systematic errors in the results. Among the sources of error mentioned were refraction, imperfect knowledge of the equation of the centre of the solar orbit, and instrumental errors.

That refraction must have influenced Ptolemy's results very considerably was already noticed by Tycho Brahe.* But I find that I underestimated the error produced by neglecting it. In the only example given by Ptolemy, the difference of longitude of sun and moon was measured on A.D. 139 February 23, "when the sun was just setting," at $5\frac{1}{2}$ equinoctial hours after Noon, while the last degree of Taurus was on the meridian. On that day the equation of time was $+15^{m}\cdot7$, so that the observation was made at $5^{h}\ 45^{m}\cdot7$ M.T. Alexandria, the sun's hour angle being $82^{\circ}\ 30'$. But a point of the ecliptic in $60^{\circ}\ 0'$ longitude or Right Ascension $57^{\circ}\ 46'$ was on the meridian. As the Sidereal Time at Mean Noon was $331^{\circ}\ 31'$, this gives the time of observation as $5^{h}\ 44^{m}$ M.T. and the sun's hour angle $82^{\circ}\ 4'\cdot5$. Of these two indications of time the former is probably by far the more reliable; we shall assume the hour angle $=82^{\circ}\ 25'$. The sun's longitude was $333^{\circ}\ 40'$ and the declination $-10^{\circ}\ 16'$.† The zenith distance was therefore $88^{\circ}\ 55'$ and the parallactic angle $q=58^{\circ}\ 0'$. The refraction in declination was $=23''\cdot8\cos q$, and in longitude

$$d\lambda = \operatorname{cosec}\epsilon \sec\lambda \cos\delta\, d\delta = 34''\cdot5.$$

By this amount, therefore, the difference of longitude of the sun and the moon, and consequently the longitude of the star, was diminished by refraction. In higher declinations of the sun, either north or south, the error would be considerably greater; in declination 20° it would be $60'$. But if the observations had been taken at sunrise, the error would have had the opposite sign, as the diminished difference of longitude of sun and moon would have increased the longitude of the star. If Ptolemy divided his observations fairly equally between sunrise and sunset, he would therefore eliminate the effect of refraction.

Similarly the error in the adopted excentricity of the sun's orbit, and consequently in the equation of the centre, may have been avoided or much diminished by distributing the observations of standard stars over the year. We know nothing whatever as to how many standard stars Ptolemy connected with the sun or how many observations of this kind he took.‡ But though errors of this kind or instrumental errors must have diminished the general accuracy of Ptolemy's star-places, the constant error of fully a degree in the longitudes cannot be accounted for in this way. The error in the equinox assumed by him can therefore only be due to an error in the mean longitude of the sun.

As mentioned in my first paper, Ptolemy adopted the elements of the solar orbit found by Hipparchus without any alteration. Whenever he quotes one of his own observations of the planets, *e.g.* of an opposition of one of the outer planets, he gives the

* *Progymnasmata*, p. 145; *Opera Omnia*, T. ii. p. 155.

† Computed from Neugebauer's Tables.

‡ He only mentions (vii. 2, p. 16, Heiberg) that he observed Spica and the brightest stars near the Ecliptic in this manner.

mean longitude of the sun at that moment.* Comparing ten of these with Neugebauer's Tables, the mean difference is $+59' \pm 0'.9$, practically the same as the constant error of Ptolemy's star-places. According to Hipparchus, the tropical year is $365\frac{1}{4} - \frac{1}{300}$ days $= 365^{d}.24666$, which is $0^{d}.00435$ too long. The mean motion of the sun in an Egyptian year of 365 days is therefore $359^{\circ}\ 45'\ 24''.8$ or $15''.7$ too small. Ptolemy's tables start from the era of Nabonassar -746 February 26, Alexandria Noon, or February 25, 22^{h} G.M.T., for which moment he gives the Mean Longitude $= 0^{\circ}\ 45'$ ♓. According to Neugebauer's Tables it was $27^{\circ}\ 56'$ ♒ or $2^{\circ}\ 49'$ less. The epoch of the star catalogue was 885 of Nabonassar, so that the correction to Ptolemy's mean longitude of the sun was $-2^{\circ}\ 49' + 15''.7 \times 885 = +1^{\circ}\ 2'$.

This, then, is the error of Ptolemy's equinox, and as it is (within a minute or two) equal to the average error of Ptolemy's longitudes of stars, it is impossible to doubt that he really founded his catalogue on new observations of stars and the sun, taking the places of the sun from the solar tables in the third book of his *Syntaxis*.

Ptolemy's veneration for his predecessor was so great, that he accepted the solar elements of Hipparchus when a few observations made by himself seemed to agree with them. He gives only the following three determinations of equinoxes made by himself:— †

Autumn Equinox	A.D.	132,	September	25,	2 p.m.,	Alex. M.T.
Autumn Equinox	,,	139,	,,	26,	7 a.m.,	,, ,,
Spring Equinox	,,	140,	March	22,	1 p.m.,	,, ,,

Computing by means of Schram's *Kalendariographische und Chronologische Tafeln* the time of entry of the sun into the signs of Libra and Aries in those years, the moments indicated by Ptolemy are found to be respectively

h	m	
32	0	too late.
31	7	,,
18	20	,,

The mean is $27^{h}\ 9^{m}$, which corresponds to an error in the mean longitude of $1^{\circ}\ 7'$. Very probably Ptolemy selected these three determinations because they agreed with the sun's motion according to Hipparchus. But it should also be remembered that such determinations of the time of equinox must have been very difficult to make. Ptolemy estimated that they may be as much as six hours in error, but the uncertainty was probably much

* It should be remembered that a planet was said to be in opposition when it was 180° from the *mean* place of the sun (simplex solis). Kepler was the first to substitute for this the true place of the sun.

† *Syntaxis*, iii. 7 and iii. 1 (ed. Heiberg, p. 256 and pp. 204–205).

greater. The observations were made by means of a large ring suspended in the plane of the equator, and the declination of the sun was supposed to be zero when the shadow of the front part of the ring fell exactly on the back part. Delambre has shown that an error of half a day or more might be caused by refraction, while the error of 13′ or 14′ in the assumed latitude of Alexandria might cause an error of about 14 hours.*

The error of Ptolemy's mean longitude of the sun did more mischief than making his stellar longitudes too small. It spoiled his determination of the constant of precession. In my paper of last year (p. 538), when speaking of the star-places found by Ptolemy for the years 295–283 B.C. and A.D. 92–98, I wrote: "By a curious piece of ill luck the longitudes for the time of Timocharis were all too great and those for the end of the first century too small, which produced the faulty precession of 1° in 100 years." But we see now that this was a necessary consequence of the error of the sun's longitude.

Year.	Mean Error of resulting Star-longitude.	Error of Sun's Mean Longitude.
−288	−38′ (4 obs.)	−50′
+96	+44′ (3 obs.)	+50′

The error in the assumed length of the tropical year is therefore responsible for the whole of the error of Ptolemy's value of Precession.

All this was clearly perceived by Laplace. In chapter ii. of the fifth book of his *Exposition du Système du Monde* † he shows that as the assumed annual motion of the sun with regard to the equinoxes was too small, the amount of precession must be increased by the arc described by the sun in an interval of time equal to the error in the length of the year. He then continues: "Ces remarques nous conduisent à examiner si, comme on le pense généralement, le catalogue de Ptolémée est celui d'Hipparque, réduit à son temps, au moyen d'une précession d'un degré dans quatre-vingt-dix ans.‡ On se fonde sur ce que l'erreur constante des longitudes des étoiles de ce catalogue, disparaît quand on le rapporte au temps d'Hipparque; mais l'explication que nous venons de donner de cette erreur, justifie Ptolémée du reproche de s'être approprié l'ouvrage d'Hipparque; et il paraît juste de l'en croire, lorsqu'il dit positivement qu'il a observé les étoiles de ce catalogue, celles même de sixième grandeur."

Ideler§ also showed how much Ptolemy's longitude of the sun was in error, and gave a table of the amount of this error. He mentions (without going into details) that Ptolemy's longitudes of stars had to be founded on his places of the sun, and that this

* *Astronomie ancienne*, ii. p. 101.

† 4th edition (1813), p. 383.

‡ That is, a degree equal to one-hundredth part of the quadrant.

§ *Historische Untersuchungen über die astronomischen Beobachtungen der Alten* (1806), p. 299. French translation by Halma (1819), p. 115.

produced the erroneous value of precession; but he does not allude to the star catalogue.

Delambre says that one might explain everything in a manner less favourable but more simple by saying that Ptolemy observed neither stars nor equinoxes but took everything from Hipparchus, starting from the minimum value of precession found by the latter.* He nowhere affirms distinctly that this is the only explanation possible; but, on the other hand, he makes the strange statement, that the error in the mean motion "n'aurait aucun inconvénient pour l'époque où il a vécu." †

In addition to the observations of stars Ptolemy records in the course of his great work more than thirty observations of the moon and the planets, and nobody seems ever to have doubted that these were really made by himself. When describing the different effect of precession in different parts of the heavens, he gives a list of the declinations of eighteen stars according to the observations of Timocharis, Hipparchus, and himself; and he remarks that they agree with the conclusion already drawn, that the stars advance in longitude at the rate of 1° in 100 years.‡ Delambre has calculated the value of the constant of precession resulting from these declinations, and finds §

Timocharis–Hipparchus . .	51″·4
Hipparchus–Ptolemy . . .	47·5
Timocharis–Ptolemy . . .	47·4

Delambre has given equal weight to all the single results, which is absurd, as some of the stars were very close to the solstitial colure (Altair, Sirius, Castor, Pollux) and the results derived from them of no value. It is very curious, that it does not strike him, that his investigation furnishes an excellent proof of the *bona fides* of Ptolemy, since the observations do *not* agree with Ptolemy's preconceived notion of 36″ a year. As some of the data used by Delambre differ from the figures in Heiberg's edition, I have repeated the calculation. I first determined the declinations by starting from those of Hipparchus, using Ptolemy's longitudes minus 1° 20′ (λ) and his latitudes unaltered (β), with his value of the obliquity $\epsilon = 23° 51'$, and his precession of 36″. We should then have

$$\Delta\delta = 160' \sin\epsilon \cos\beta \cos\lambda \sec\delta.$$

Or, we may, like Delambre, from this formula find what annual change of longitude corresponds to the change of declination from Hipparchus to Ptolemy.

* *Astronomie ancienne*, ii. p. 250; see also i. p. 183.
† *Ibid.*, ii. p. 138.
‡ *Syntaxis*, vii. 3 (Heiberg, p. 19 *sq.*).
§ *Astronomie ancienne*, ii. p. 254.

	$\Delta\delta$.	Declination. Computed with 36″.	Declination. Ptolemy.	Calc.-Obs.	Precession from $d\delta$ Observed.
	′	° ′	° ′	′	″
α Aquilæ	+ 2·5	+ 5 50	+ 5 50	0	29·2
Alcyone	+57·2	+16 7	+16 15	− 8	41·2
Aldebaran	+49·2	+10 34	+11 0	−26	55·3
Capella	+46·9	+41 11	+41 10	+ 1	35·6
γ Orionis	+37·4	+ 2 25	+ 2 30	− 5	40·6
α Orionis	+30·4	+ 4 50	+ 5 15	−25	65·5
Sirius	+12·3	−15 48	−15 45	− 3	44·1
Castor	+10·6	+33 21	+33 24	− 3	47·7
Pollux	+ 6·1	+30 6	+30 10	− 4	59·9
Regulus	−35·7	+20 4	+19 50	+14	50·7
Spica	−64·5	− 0 29	− 0 30	+ 1	37·1
η Ursæ	−64·8	+59 40	+59 40	0	36·4
ζ Ursæ	−65·0	+65 25	+65 0	+25	50·2
ε Ursæ	−64·5	+66 31	+66 15	+16	45·5
Arcturus	−63·9	+29 56	+29 50	+ 6	39·7
α Libræ	−62·4	− 6 38	− 7 10	+32	55·8
β Libræ	−59·8	− 0 36	− 1 0	+24	50·9
Antares	−51·5	−19 52	−20 15	+23	52·7

Most of the results in the last column differ very little from Delambre's. Omitting the four stars mentioned above, the mean is 46″·9. The second last column also shows that the declinations given by Ptolemy agree very badly with his precessional constant of 36″. They must therefore have been observed, and cannot be those of Hipparchus reduced to A.D. 137. I may add that Ptolemy's knowledge of spherical trigonometry was quite sufficient to enable him to do that, if he had wished.*

If Ptolemy had possessed one of our modern transit-circles, and had allowed for refraction, etc., but had taken the places of the sun from his own tables, his star-places might have represented the relative positions of the stars as accurately as one of the modern catalogues does. But they would certainly all the same have been affected with the large systematic error which we find in his Catalogue. And when in the nineteenth century a faulty determination of the equinox of 1865 produced a value of the constant of precession (that of Nyrén) which differed in an extraordinary manner from those found by other astronomers, it was only a repetition, on a different scale, of what had happened in the second century.

The assertion that Ptolemy's Catalogue is only a reproduction of the catalogue of Hipparchus was (I believe) first made by Tycho

* *Cf.* Braunmühl, *Geschichte der Trigonometrie*, i. p. 23 *sq.*, and Zeuthen's paper, "Note sur la trigonometrie de l'antiquité," in *Bibliotheca Mathematica* I_3 (1900), pp. 20–27.

Brahe.* Repeated by Flamsteed in the introduction to his *Historia Cœlestis,* and further circulated by Delambre, it has been accepted more or less as a self-evident proposition, while the explanation given by Laplace and Ideler has been overlooked. This would hardly have happened if all but one of the writings of Hipparchus had not been lost. The only book that is left, the *Commentary to Aratus,* was written before the discovery of precession was made. The remarkable catalogue of the Right Ascensions of 44 time-stars contained in that book, and the Declinations quoted by Ptolemy, show that he observed these co-ordinates directly, at least for some stars; and it is quite possible, as suggested by Delambre,† that the Catalogue of Hipparchus originally gave the Right Ascension and Declination of Stars, from which the Longitudes and Latitudes were derived by calculation after the discovery of precession. On the other hand, it seems from what Ptolemy says of the observations of the moon by Hipparchus, ‡ that these were made with an instrument which gave longitude and latitude directly, and he may of course also have observed fixed stars with it.

The Atomic Weights of the Elements in Nebulæ. By Professor J. W. Nicholson, F.R.S.

The interpretations, which have been put forward by the writer in various papers in the *Monthly Notices,* of the nebular and coronal spectra in terms of the dynamical vibrations of certain precisely defined atoms about stationary states of steady motion, have hitherto been deficient in one sense at least. A completely defined atom should be capable of an investigation which gives the relative wave-lengths of lines with any desired order of precision, although, on the electromagnetic basis, such an investigation is so complex that the problem, as treated in these papers, was simplified so that the forces acting on the electrons of the atom, during their vibrations, were regarded as purely of the electrostatic type. It remains to be shown that, in the more complete investigation, in which the necessary magnetic and associated mechanical forces on any electron due to its motion are taken into account, the numerical results obtained are a sufficiently close approximation to the truth. The main object of this paper is to obtain a second approximation to the true periods of vibration of a single-ring atom, and thence to deduce, as accurately as possible, the atomic weight of the element designated as nebulium. Bourget, Buisson, and Fabry's experimental investigation, by use of the limiting order of interference which can be obtained from the line λ 5007, has

* *Progymnasmata,* p. 138 (*Opera,* ii. p. 151), and introduction to Catalogue of 1000 Stars, Gassendi Vita Tychonis (Hague edition, p. 248), *Opera,* iii. p. 335.

† *Astronomie ancienne,* i. pp. 117, 172, 184.

‡ *Syntaxis,* v. 5 (Heiberg, pp. 369, 375).

already shown qualitatively that the atomic weight of the element producing this line is between those of hydrogen and helium.

Recalling the results of previous papers, which depend on the electrostatic theory alone, we note that the two most fundamental —and necessarily stable—vibrations of an atom, consisting, in its stationary state, of a ring of n electrons each of mass m and charge $-e$, rotating round a nucleus of change ve and much greater mass M with angular velocity ω, have periods $2\pi/(q_1, q_2)$, where $q_1 = \omega$ and

$$\frac{q_2^2}{\omega^2} = \frac{\nu}{\nu - \frac{1}{4}s_n}\left(1 + n\frac{m}{\mathrm{M}}\right),$$

s_n denoting $\sum_{r=1}^{n-1}$ cosec $r\pi/n$. The second is a transverse vibration of the ring as a whole. The ratio of the corresponding wave-lengths in the spectrum is

$$\left(\frac{\nu}{\nu - \frac{1}{4}s_n}\right)^{\frac{1}{3}}\left(1 + n\frac{m}{\mathrm{M}}\right)^{\frac{1}{3}},$$

where, if n and ν are prescribed, the ratio is completely determined if the ratio m/M is known, and conversely, if the lines have known wave-lengths of sufficient experimental accuracy, m/M can be determined, and thence the atomic weight of the element. The accuracy attained by the electrostatic method, and thence by this equation, is dependent on the fact that the linear velocity of the electrons in their steady orbit is only of magnitude about $4{\cdot}10^{-3}\,c$, where c is the velocity of light. In heavier atoms this velocity must be supposed to be greater, and the investigation below may in such future cases, if vibrational spectra from such atoms are obtained, take on an even greater degree of significance. At present it serves to justify the omissions in previous papers.

Inclusion of the Magnetic Effects of Motion.

Schott* has given the following formula for the accurate period equation of transverse vibrations of a ring of n electrons, forming a Saturnian system around a stationary nucleus ve :—

Let
$$\mathrm{J} = \tfrac{1}{4}\textstyle\sum_1^{n-1} \sin^2 k\pi s/n \operatorname{cosec}^3 \pi s/n\,,$$
$$\mathrm{K} = \tfrac{1}{4}\textstyle\sum_1^{n-1} \operatorname{cosec} s\pi/n = \tfrac{1}{4}s_n\,,$$
$$\mathrm{H} = \tfrac{1}{4}\textstyle\sum_1^{n-1} \sin^2 k\pi s/n \operatorname{cosec} \pi s/n\,,$$
$$\mathrm{M} = \tfrac{1}{8}\textstyle\sum_1^{n-1} \sin^2 ks\pi/n \cos s\pi/n \operatorname{cosec}^2 s\pi/n\,,$$
$$m = 2s + 1\,, \quad l = 2q/\omega + 2s + 1\,,$$

$$f(k, q) = \sum_{s=-\infty}^{\infty} \tfrac{1}{1^6} \cot(2s - 2k + 1)\frac{\pi}{2n}\Big[-(1-\beta^2)l^2\beta^2 \mathrm{J}_m'(l\beta) + (1+3\beta^2)l\beta \mathrm{J}_m(l\beta)$$
$$+ \{(1+\beta^2)(m^2 + l^2\beta^2) - 4ml\beta^2 - 1 - 3\beta^2\} \int_0^{l\beta} \mathrm{J}_m(y)dy \Big],$$

where the functions J_m are Bessel functions.

* *Phil. Mag.*, 1907 Feb., p. 210.

Then the period equation for axial vibrations is

$$\nu-(1-\beta^2)^2\mathrm{J}-\tfrac{1}{2}\beta^2(5-\beta^2)\mathrm{H}=f(0,0)-f(k,q)-2(1-\beta^2)\mathrm{M}\frac{q\beta^2}{\omega}+\left\{(1+\beta^2)(\mathrm{K}-2\mathrm{H})+\frac{c^2m\rho}{e^2}\right\}\left(\frac{q\beta}{\omega}\right)^2,$$

where ρ is the radius of the orbit. The corresponding accurate equation of steady motion, corresponding to Kepler's law, is

$$m\rho\omega^2=\frac{e^2}{\rho^2}(\mathrm{V}-\mathrm{K}(1+\beta^2))+\frac{\nu e^2}{\rho^2},$$

where

$$\mathrm{V}=\sum_{s=0}^{\infty}(s+\tfrac{1}{2})\cot(2s+1)\frac{\pi}{2n}\Big[\beta(1-\beta^2)\mathrm{J}_{2s+1}\{(2s+1)\beta\}+(1+\beta^2)\int_0^{\beta}\mathrm{J}_{2s+1}\{(2s+1)x\}dx\Big]$$

and $\mathrm{K}=\frac{1}{4}s_n$.

Schott has not incorporated in his recent treatise,* which sums up and extends these results of his separate papers, any proof of the above period equation, but an investigation by his method, which the writer has undertaken, confirms the result. This investigation is somewhat laborious, and it was not thought desirable to include it in this communication.

In seeking only a second approximation to the periods, in which β, the ratio of the steady velocity of an electron to that of light, is very small (of the order of magnitude already suggested), it is only necessary to consider the first few terms of V, which is an expression converging very rapidly for small values of β. We include more terms than are really necessary, for possible use later.

Thus we write

$$\mathrm{J}_1(\beta)=\frac{\beta}{2}\left(1-\frac{\beta^2}{2^2\,1!\,2}+\frac{\beta^4}{2^4\,2!\,2.3}+\ldots\right)=\frac{1}{2}\beta\left\{1-\frac{\beta^2}{8}+\frac{\beta^4}{192}\right\},$$

$$\mathrm{J}_3(\beta)=\frac{\beta^3}{2^3\,3!}\left(1-\frac{\beta^2}{2^2\,1!\,4}\right)=\frac{\beta^3}{48}\left(1-\frac{\beta^2}{16}\right),$$

$$\mathrm{J}_5(\beta)=\frac{\beta^5}{2^5\,4!}=\frac{\beta^5}{768},$$

and

$$\mathrm{V}=\frac{1}{2}\cot\frac{\pi}{2n}\left\{(\beta-\beta^3)\frac{\beta}{2}\left(1-\frac{\beta^2}{8}\right)+\frac{(1+\beta^2)}{2}\cdot\left(\frac{\beta^2}{2}-\frac{\beta^4}{32}\right)\right\}+\frac{3}{2}\cot\frac{3\pi}{2n}\left\{(\beta-\beta^3)\cdot\frac{(3\beta)^3}{2^3\,3!}+\frac{(1+\beta^2)3^3}{2^3\,3!}\frac{\beta^4}{4}\right\},$$

$$=\frac{1}{4}\cot\frac{\pi}{2n}\left\{\beta^2-\frac{\beta^4}{8}-\beta^4+\frac{\beta^2}{2}-\frac{\beta^4}{32}+\frac{\beta^4}{2}\right\}+\frac{3}{2}\cot\frac{3\pi}{2n}\cdot\frac{3^3}{2^3\,3!}\left\{\beta^4+\frac{\beta^4}{4}\right\},$$

$$=\frac{1}{4}\cot\frac{\pi}{2n}\left\{\frac{3}{2}\beta^2-\frac{21}{32}\beta^4\right\}+\frac{27}{32}\cdot\frac{5}{4}\beta^4\cot\frac{3\pi}{2n}.$$

* *Electromagnetic Radiation*, Cambridge University Press.

To a second approximation, the equation of steady motion is

$$\omega^2 = \frac{e^2}{m\rho^3}\left\{\nu - \frac{1}{4}s_n + \beta^2\left\{-\frac{1}{4}s_n + \frac{3}{8}\cot\frac{\pi}{2n}\right\}\right\},$$

and this equation at least is only affected to an order β^2.

Returning to the vibrations, we note that for the symmetrical and most stable one of class zero, as discussed in several papers and corresponding to $k=0$,

$$f(k,q)=f(0,q), \quad m=2s+1, \quad l=2q/\omega+2s+1,$$

$$J_m(l\beta)=\frac{(l\beta)^m}{2^m\Gamma(m+1)}=\frac{(l\beta)^{2s+1}}{2^{2s+1}2s!} \text{ if } s \text{ is positive,}$$

$$J_m(l\beta)=\frac{(-)^m(l\beta)^m}{2^m\Gamma(m+1)}=-\frac{(l\beta)^{2s-1}}{2^{2s-1}2s-1!} \text{ if } s \text{ is negative,}$$

to first approximations.

The term corresponding to $s=0$ in $f(0, q)$ is

$$\frac{1}{16}\cot\frac{\pi}{2n}\Big[-(1-\beta^2)l^2\beta^2J_1'(l\beta)+(1+3\beta^2)l\beta J_1(l\beta) + \{(1+\beta^2)(m^2+l^2\beta^2)-4ml\beta^2-1-3\beta^2\}\int_0^{l\beta}J_1(y)dy\Big],$$

or, retaining only the first two significant powers of β, the convergence of the expanded series being similar to that of the earlier series, this becomes

$$\frac{1}{16}\cot\frac{\pi}{2n}\Big[2l^2\beta^4-\tfrac{1}{4}l^4\beta^4+\frac{\beta^4}{4}(l^4-4l^3-2l^2)\Big] = \frac{l^2\beta^4}{16}\cot\frac{\pi}{2n}\left(\frac{3}{2}-l\right)=\frac{\beta^4}{16}\cot\frac{\pi}{2n}\cdot\left(\frac{2q}{\omega}+1\right)^2\left(\frac{1}{2}-\frac{2q}{\omega}\right)$$

of order β^4.

The term corresponding to $s=1$ is

$$\frac{1}{16}\cot\frac{3\pi}{2n}\Big[-(1-\beta^2)l^2\beta^2J_3'(l\beta)+(1+3\beta^2)l\beta J_3(l\beta) + \{(1+\beta^2)(m^2+l^2\beta^2)-4ml\beta^2-1-3\beta^2\}\int_0^{l\beta}J_3(y)dy\Big],$$

and is also evidently at least of order β^4. Its first approximation becomes

$$\frac{1}{16}\cot\frac{3\pi}{2n}\Big[-\frac{l^4\beta^4}{16}+\frac{l^4\beta^4}{48}+(m^2-1)\frac{l^4\beta^4}{192}\Big]=0,$$

since $m=3$ in the case $s=1$. The term is therefore of order β^6.

The term corresponding to $s=-1$ is

$$-\frac{1}{16}\cot\frac{\pi}{2n}\Big[+(1-\beta^2)l^2\beta^2J_1'(l\beta)-(1+3\beta^2)l\beta J_1(l\beta) + \{(1+\beta^2)(m^2+l^2\beta^2)-4ml\beta^2-1-3\beta^2\}\int_0^{l\beta}-J_1(y)dy\Big],$$

or, to an order β^4, the terms in β^2 again disappearing,

$$-\frac{l^2\beta^4}{16}\left(\frac{3}{2}-l\right)\cot\frac{\pi}{2n},$$

or, since $l=2q/\omega-1$ in this case, we obtain

$$-\frac{\beta^4}{16}\cot\frac{\pi}{2n}\cdot\left(\frac{2q}{\omega}-1\right)^2\left(\frac{5}{2}-2q/\omega\right).$$

All other terms of the series for s positive are at least of order β^6, by inspection. When s is less than -2, this also occurs. If $s=-2$, the most important part of the corresponding term is

$$-\frac{1}{16}\cot\frac{3\pi}{2n}\left\{+l^2\beta^2\mathrm{J}_3'(l\beta)-l\beta\mathrm{J}_3(l\beta)-(m^2-1)\int_0^\beta \mathrm{J}_3(y)dy\right\}$$

$$=-\frac{1}{16}\cot\frac{3\pi}{2n}\left\{\frac{l^4\beta^4}{16}-\frac{l^4\beta^4}{48}-(m^2-1)\frac{l^4\beta^4}{192}\right\}=0,$$

since $m=2s+1=-3$. It is therefore of order β^6.

Finally, therefore, to order β^6 exclusive,

$$f(0,q)=\frac{\beta^4}{16}\cot\frac{\pi}{2n}\left\{\left(\frac{2q}{\omega}+1\right)^2\left(\frac{1}{2}-\frac{2q}{\omega}\right)-\left(\frac{2q}{\omega}-1\right)^2\left(\frac{5}{2}-\frac{2q}{\omega}\right)\right\},$$

$$f(0,0)=\frac{\beta^4}{16}\cot\frac{\pi}{2n}\cdot\left(\frac{1}{2}-\frac{5}{2}\right),$$

$$\therefore\ f(0,0)-f(0,q)=\frac{\beta^4}{16}\cdot\frac{12q}{\omega}\left(1-\frac{2q}{\omega}\right)\cot\frac{\pi}{2n}$$

$$=\frac{3}{4}\beta^4\frac{q}{\omega}\left(1-\frac{2q}{\omega}\right)\cot\frac{\pi}{2n}.$$

The period equation has therefore the second approximation,

$$\nu-\mathrm{J}(1-2\beta^2)-\frac{5}{2}\beta^2\mathrm{H}=-2\beta^2\frac{q}{\omega}\mathrm{M}+\left[(1+\beta^2)(\mathrm{K}-2\mathrm{H})+\frac{c^2m\rho}{e^2}\right]\left(\frac{q\beta}{\omega}\right)^2.$$

But

$$\frac{c^2m\rho}{e^2}=\frac{c^2}{\rho^2\omega^2}\cdot\frac{\omega^2m\rho^3}{e^2}$$

$$=\frac{1}{\beta^2}\left\{\nu-\frac{1}{4}s_n+\beta^2\left[-\frac{1}{4}s_n+\frac{3}{8}\cot\frac{\pi}{2n}\right]\right\}$$

by the equation of steady motion.

But for vibrations of class zero

$$\mathrm{J}=\mathrm{M}=\mathrm{H}=0,$$

each of these quantities vanishing with k, the class of the vibration.

Thus

$$\nu = \frac{q^2}{\omega^2}\left\{\nu - \frac{1}{4}s_n + \mathrm{K}\beta^2 + \beta^2\left(-\frac{1}{4}s_n + \frac{3}{8}\cot\frac{\pi}{2n}\right)\right\},$$

and as $\mathrm{K} = \frac{1}{4}s_n$,

$$\frac{q}{\omega} = \sqrt{\nu\Big/\left(\nu - \frac{1}{4}s_n + \frac{3}{8}\cot\frac{\pi}{2n}\,.\,\beta^2\right)},$$

differing to the second order of β, for a stationary nucleus, from the value previously deduced.

In the case of the neutral atom of nebulium we found $\beta = 0{\cdot}0043$, $\frac{3}{8}\beta^2 = 0{\cdot}0^5693$, so that

$$\frac{3}{8}\cot\frac{\pi}{2n}\beta^2 = 0{\cdot}00002,$$

and the value of q/ω, uncorrected for the nucleus, is not altered to five figures.

Corrected Vibrations in the Plane.

We now enter upon the difficult problem of a second approximation to the vibrations in the plane of the atom, with a view to an accurate value of m/M for an atom of nebulium. The formula for such vibrations has not been formally derived by Schott, but the greater part of the necessary analysis may be found in his papers.

We shall only consider the vibration of class zero, for which the approximate formula is $q = \omega$. It is essential to the conclusions of earlier papers that no correction of order β should be found in this case. In the presence of such a correction, the value of m/M would be greatly changed.

Schott has given the following equations of disturbed motion of a ring of electrons, and the results have been independently verified subsequently by the writer. The equations quoted here are simplified so as to include only a disturbance in the plane. The more general forms are to be found in the *Phil. Mag.*, 1908 January, pp. 180–183, where the notation is explained.

If η is the tangential displacement of an electron, and ξ is the displacement along the *inward* radius of the ring,

$$\mu\ddot{\xi} - (\mu + m)\omega\dot{\eta} = \delta\tau,$$
$$(\mu + m)\omega\dot{\xi} + m\ddot{\eta} - \mu\omega^2\eta = \delta\mathrm{P},$$

where m is the *transverse* mass of an electron, and μ is the *longitudinal* mass derived according to the formula

$$\mu = d(m\beta)/d\beta.$$

$\delta\tau$ and $\delta\mathrm{P}$ are the components of mechanical force on an electron due (1) to the controlling field, in the present case consisting only of the positive nucleus in a stationary position; (2) to the action of the other electrons of the ring on the electron whose motion is considered. They are the increments from steady motion values.

Certain terms, due to radiation, which appear on the left side of the equations are already cancelled with some equal terms in the expressions for δP, $\delta\tau$. The uncancelled terms of δP, $\delta\tau$, in so far as they are due to the other electrons in the ring, are given by

$$\frac{\rho^3}{e^2}\delta\tau = -\xi\left[(1-\beta^2)N - 4M\beta^2\frac{q}{\omega} - (2K-4H)\beta^2\frac{q^2}{\omega^2} + \bar{A} - \bar{A}_0\right] + \iota\eta\left[(1+\beta^2)M + (2K-2H+W)\beta^2\frac{q}{\omega} - \bar{B}\right],$$

$$\frac{\rho^3}{e^2}\delta P = -\iota\xi\left[(1+\beta^2)M + (2K-2H+W)\beta^2\frac{q}{\omega} - \bar{B}\right] + \eta\left[\frac{(1-\beta^2)^2}{2}N + \frac{3+\beta^2}{2}H - 2K + 2V - \beta\frac{\partial V}{\partial\beta} - \bar{C} + \bar{C}_0 - 2M(1-\beta^2)\beta^2\frac{q}{\omega} + (K-2H)(1+\beta^2)\beta^2\frac{q^2}{\omega^2}\right].$$

Of the various functions contained in these formulæ, the following three,

$$H = \frac{1}{4}\sum_{s-1}^{n-1}\operatorname{cosec}\frac{s\pi}{n}\sin^2\frac{ks\pi}{n},$$

$$M = \frac{1}{8}\sum_{1}^{n-1}\cot\frac{s\pi}{n}\operatorname{cosec}\frac{s\pi}{n}\sin 2ks\frac{\pi}{n},$$

$$N = \sum_{1}^{n-1}\left(\frac{1}{2}\operatorname{cosec}^3\frac{s\pi}{n} - \frac{1}{4}\operatorname{cosec}\frac{s\pi}{n}\right)\sin^2\frac{ks\pi}{n}$$

vanish with k, and are therefore zero for the vibration of class zero. The formulæ are therefore simplified for our present purpose to

$$\frac{\rho^3}{e^2}\delta\tau = -\xi\left[\bar{A} - \bar{A}_0 - \tfrac{1}{2}s_n\beta^2\frac{q^2}{\omega^2}\right] + \iota\eta\left[(\tfrac{1}{2}s_n + W)\beta^2\frac{q}{\omega} - \bar{B}\right],$$

$$\frac{\rho^3}{e^2}\delta P = -\iota\xi\left[(\tfrac{1}{2}s_n + W)\beta^2\frac{q}{\omega} - \bar{B}\right] + \eta\left[\tfrac{1}{4}s_n(1+\beta^2)\beta^2\frac{q^2}{\omega^2} + \bar{C}_0 - \bar{C} + 2V - \beta\frac{\partial V}{\partial\beta} - \frac{1}{2}s_n\right],$$

where ρ is the radius of the orbit, and K has been written $\frac{1}{4}s_n$, in accordance with our usual notation in earlier papers. We proceed to discuss the other functions in detail. In the first place,

$$W = \sum_{s=0}^{\infty}(s+\tfrac{1}{2})\cot(2s+1)\frac{\pi}{2n}\left[\beta J_{2s+1}(\overline{2s+1}\beta) - \int_0^\beta J_{2s+1}\{(2s+1)x\}dx\right].$$

The terms rapidly rise in order of β. The term $s=0$ becomes

$$\frac{1}{2}\cot\frac{\pi}{2n}\left\{\frac{\beta^2}{2}\left(1-\frac{\beta^2}{8}\right) - \frac{1}{2}\left(\frac{\beta^2}{2} - \frac{\beta^4}{32}\right)\right\} = \frac{\beta^2}{8}\cot\frac{\pi}{2n}$$

to the second order.

The term $s=1$ gives

$$\frac{3}{2}\cot\frac{3\pi}{2n}\left[\frac{9}{16}\beta^4-\frac{9}{64}\beta^4\right],$$

and may be neglected with all higher terms. Thus

$$W=\frac{\beta^2}{8}\cot\frac{\pi}{2n},$$

and as W is always multiplied by β^2 in the formulæ for $\delta\tau$, δP, its contributions are of the *fourth* order only.

Secondly, the function V is defined (Schott, *l.c.*, p. 174) by

$$V=\sum_{s=0}^{\infty}(s+\tfrac{1}{2})\cot(2s+1)\frac{\pi}{2n}\left[\beta(1-\beta^2)J_{2s+1}\{\overline{2s+1}\beta\}\right.$$
$$\left.+(1+\beta^2)\int_0^{\beta}J_{2s+1}(\overline{2s+1}\beta)d\beta\right],$$

and is, in fact, the function which occurs in the accurate equation of steady motion, of the section immediately preceding. Its value to the second order of β is

$$V=\frac{3}{8}\beta^2\cot\frac{\pi}{2n}.$$

Thus

$$2V-\beta\frac{\partial V}{\partial\beta}=0$$

to the second order, and is, in fact, of the fourth order in β.

Thus the mechanical forces are further simplified to

$$\frac{\rho^3}{e^2}\delta=-\xi\left\{\bar{A}-\bar{A}_0-\frac{1}{2}s_n\beta^2\frac{q^2}{\omega^2}\right\}+\iota\eta\left\{\frac{1}{2}s_n\beta^2\frac{q}{\omega}-\bar{B}\right\},$$

$$\frac{\rho^3}{e^2}\delta P=-\iota\xi\left\{\frac{1}{2}s_n\beta^2\frac{q}{\omega}-\bar{B}\right\}+\eta\left\{\frac{1}{4}s_n\beta^2\frac{q^2}{\omega^2}+\bar{C}_0-\bar{C}-\frac{1}{2}s_n\right\},$$

and these are correct to the *third* order of β inclusive.

The functions $\bar{A}$, B, $\bar{C}$ are given by

$$\bar{A}=\sum_{s=-\infty}^{\infty}\cot(2s+1)\frac{\pi}{2n}.\ a\left\{2k+2s+1,\left(2k+2s+1+\frac{2q}{\omega}\right)\beta\right\},$$

with similar expressions for $\bar{B}$, $\bar{C}$, where

$$a(m,l\beta)=\frac{l^2\beta^2}{4}J_m'(l\beta)-\frac{m^2-l^2\beta^2}{8}.\ l\int_0^{\beta}J_m(lx)dx,$$

$$b(m,l\beta)=\frac{m-l\beta^2}{8}l\beta J_m(l\beta)+\frac{m+l\beta^2}{8}.\ l\int_0^{\beta}J_m(lx)dx,$$

$$c(m,l\beta)=\frac{1-\beta^2}{16}l^2\beta^2J_m'(l\beta)+\frac{3+\beta^2}{16}l\beta J_m(l\beta)$$
$$+\frac{1}{16}[(1+\beta^2)(m^2+l^2\beta^2-4ml\beta^2-3-\beta^2)]l\int_0^{\beta}J_m(lx)dx,$$

where, as in the transverse vibrations,

$$m = 2s + 1, \quad l = \frac{2q}{\omega} + 2s + 1.$$

The lowest terms in these latter formulæ, *which are all even functions of* β, are found, when m is not negative, for the value $m = 1$, corresponding to $s = 0$ in the summations, and become

$$a(m, l\beta) = \frac{l^2\beta^2}{4} \cdot \frac{1}{2} - \frac{m^2 l^2}{8} \cdot \frac{\beta^2}{4} = \frac{3}{32} l^2\beta^2 \qquad (s = 0),$$

$$b(m, l\beta) = \frac{ml\beta}{8} \cdot \frac{l\beta}{2} + \frac{m}{8} \cdot \frac{l^2\beta^2}{4} = \frac{3}{32} l^2\beta^2 \qquad (s = 0),$$

$$c(m, l\beta) = \frac{l^2\beta^2}{16} \cdot \frac{1}{2} + \frac{3l\beta}{16} \cdot \frac{l\beta}{2} + \frac{1}{16} \cdot (m^2 - 3) \cdot \frac{1}{4} l^2\beta^2$$

$$= \frac{3}{32} l^2\beta^2 \qquad (s = 0),$$

all the three values being equal. *In these formulæ,* $l = (2q/\omega + 1)$. When m is negative, $s = -1$ or $m = -1$, which corresponds to it, is the only case giving terms in β^2. These terms are

$$a(m, l\beta) = -\frac{3}{32} l^2\beta^2 \qquad (s = -1),$$

$$b(m, l\beta) = +\frac{3}{32} l^2\beta^2 \qquad (s = -1),$$

$$c(m, l\beta) = -\frac{3}{32} l^2\beta^2 \qquad (s = -1),$$

and in these formulæ $l = (2q/\omega - 1)$.

Thus to order β^2, we finally obtain

$$\bar{A} = \frac{3}{32} \beta^2 \cot \frac{\pi}{2n} \cdot \left\{ \left(\frac{2q}{\omega} + 1 \right)^2 + \left(\frac{2q}{\omega} - 1 \right)^2 \right\}$$

$$= \frac{3}{16} \beta^2 \cot \frac{\pi}{2n} \cdot \left\{ \frac{4q^2}{\omega^2} + 1 \right\},$$

$$\bar{C} = \frac{3}{16} \beta^2 \cot \frac{\pi}{2n} \left\{ \frac{4q^2}{\omega^2} + 1 \right\},$$

$$\bar{B} = \frac{3}{32} \beta^2 \cot \frac{\pi}{2n} \cdot \left\{ \left(\frac{2q}{\omega} + 1 \right)^2 - \left(\frac{2q}{\omega} - 1 \right)^2 \right\}$$

$$= \frac{3}{4} \beta^2 \frac{q}{\omega} \cot \frac{\pi}{2n}.$$

The suffixed letters are the corresponding values with q equated to zero, and become

$$\bar{A}_0 = \bar{C}_0 = \frac{3}{16} \beta^2 \cot \frac{\pi}{2n}, \qquad \bar{B}_0 = 0,$$

and finally,

$$\bar{A} - \bar{A}_0 = \frac{3}{4}\beta^2 \frac{q^2}{\omega^2} \cot \frac{\pi}{2n} = \bar{C} - \bar{C}_0,$$

$$\bar{B} = \frac{3}{4}\beta^2 \frac{q}{\omega} \cot \frac{\pi}{2n},$$

and, to order β^3, the mechanical forces exerted on any one electron by the others in its ring may be identified with the results of

$$\frac{\rho^3}{e^2}\delta\tau = -\frac{\xi}{2}\left\{\frac{3}{2}\cot\frac{\pi}{2n} - s_n\right\}\beta^2\frac{q^2}{\omega^2} + \frac{\iota\eta}{2}\left\{s_n - \frac{3}{2}\cot\frac{\pi}{2n}\right\}\beta^2\frac{q}{\omega},$$

$$= -\left(\frac{3}{4}\cot\frac{\pi}{2n} - \frac{1}{2}s_n\right)\beta^2\frac{q}{\omega}\left(\frac{q\xi}{\omega} + \iota\eta\right),$$

$$\frac{\rho^3}{e^2}\delta P = -\frac{\xi\iota}{2}\left\{s_n - \frac{3}{2}\cot\frac{\pi}{2n}\right\}\beta^2\frac{q}{\omega} + \frac{\eta}{4}\left(s_n - 3\cot\frac{\pi}{2n}\right)\frac{\beta^2 q^2}{\omega^2} - \frac{\eta}{2}s_n.$$

The mechanical forces due to the nucleus must be added, *i.e.* the increments from the steady motion values. The mutual potential energy of an electron and the nucleus is $-e\Omega$, if Ω is the potential of the nucleus at an external point, or,

$$\Omega = \frac{\nu e}{r}.$$

Thus the tangential force is of course zero, and the force along the *inward* normal, tending to increase r, is

$$+\frac{\partial}{\partial r}\left(-\frac{\nu e^2}{r}\right) = \frac{\nu e^2}{r^2}.$$

If $r = \rho - \eta$, the increment from the steady value is

$$\frac{2\nu e^2 \eta}{\rho^3},$$

and the final equations of motion become

$$\mu\ddot{\xi} - (\mu + m)\omega\dot{\eta} = -\frac{\beta^2}{\rho^3}\frac{\dot{q}e^2}{\omega}\left(\frac{q\xi}{\omega} + \iota\eta\right)\left(\frac{3}{4}\cot\frac{\pi}{2n} - \frac{1}{2}s_n\right),$$

$$(\mu + m)\omega\dot{\xi} + m\ddot{\eta} - \mu\omega^2\eta = -\frac{\iota\xi e^2}{2\rho^3}\cdot\beta^2\frac{q}{\omega}\left(s_n - \frac{3}{2}\cot\frac{\pi}{2n}\right)$$

$$+\frac{e^2\eta}{4\rho^3}\cdot\frac{\beta^2 q^2}{\omega^2}\left(s_n - 3\cot\frac{\pi}{2n}\right) - \frac{\eta s_n e^2}{2\rho^3} + \frac{2\nu e^2\eta}{\rho^3}.$$

We may first test the equations in their first approximation, for which the longitudinal and transverse masses of the electron are equal. Thus

$$m(\ddot{\xi} - 2\omega\dot{\eta}) = 0,$$

$$m(2\omega\dot{\xi} + \ddot{\eta} - \omega^2\eta) = \frac{2e^2}{\rho^3}\eta\left(\nu - \frac{1}{4}s_n\right) = 2m\omega^2\eta,$$

or if ξ and η are proportional to $e^{\iota q t}$,

$$q\xi + 2\iota\omega\eta = 0,$$
$$2\omega\iota q\xi - (q^2 + \omega^2)\eta = 2\omega^2\eta,$$

therefore
$$(q^2 + 3\omega^2)q^2 = 4\omega^2 q^2,$$

or
$$q = \pm\omega,$$

which is the proper solution to this order, and, in fact, the principal vibration of the atom already quoted in earlier papers.

Final Solution.

In obtaining a solution to the required order, it is necessary to specify the type of electron which is imagined, in order to fix the relation between the longitudinal and transverse mass. We shall leave this specification until the end, in order to obtain a result appropriate to any type of electron. All mass-formulæ hitherto proposed give equal longitudinal and transverse masses for slow speeds, so that the specification is not necessary in the first approximations already dealt with in earlier papers.

With the revised equation of steady motion,

$$\omega^2 = \frac{e^2}{m\rho^3}\left\{\nu - \frac{1}{4}s_n + \beta^2\left(-\frac{1}{4}s_n + \frac{3}{8}\cot\frac{\pi}{2n}\right)\right\},$$

and writing

$$\frac{\partial}{\partial t} \equiv \iota q, \quad \frac{\partial^2}{\partial t^2} \equiv -q^2,$$

the equations of motion become

$$\mu q^2\xi + (\mu + m)\iota q\omega\eta = \frac{\beta^2 q}{\omega}\cdot\frac{e^2}{\rho^3}\left(\frac{q\xi}{\omega} + \iota\eta\right)\left(\frac{3}{4}\cot\frac{\pi}{2n} - \frac{1}{2}s_n\right),$$

$$(\mu + m)\iota q\omega\xi - \eta(mq^2 + 2m\omega^2 + \mu\omega^2) = -\frac{\iota\beta^2 q\xi}{2\omega}\frac{e^2}{\rho^3}\left(s_n - \frac{3}{2}\cot\frac{\pi}{2n}\right)$$
$$+ \frac{e^2\eta\beta^2}{4\rho^3}\left\{s_n\left(2 + \frac{q^2}{\omega^2}\right) - \left(\frac{q^2}{\omega^2} + 1\right)3\cot\frac{\pi}{2n}\right\}.$$

It is, moreover, lawful to write $q = \omega$ in terms involving β^2, so that, if $\mu/m = \alpha$,

$$\alpha q^2\xi + (1 + \alpha)\iota q\omega\eta = \frac{e^2}{m\rho^3}\beta^2(\xi + \iota\eta)\left(\frac{3}{4}\cot\frac{\pi}{2n} - \frac{1}{2}s_n\right),$$

$$(1 + \alpha)\iota q\omega\xi - \eta(q^2 + 2\omega^2 + \alpha\omega^2) = +\frac{\iota e^2}{m\rho^3}\beta^2\xi\left(\frac{3}{4}\cot\frac{\pi}{2n} - \frac{1}{2}s_n\right)$$
$$+ \frac{3}{4}\frac{e^2}{m\rho^3}\eta\beta^2\left(s_n - 2\cot\frac{\pi}{2n}\right),$$

for which we derive a period equation of the form (substituting the first approximation

$$\omega^2 = \frac{e^2}{m\rho^3}\left(\nu - \frac{1}{4}s_n\right)$$

in the terms involving β^2)

$$\left\{\alpha q^2 - \frac{\beta^2\omega^2\left(\frac{3}{4}\cot\frac{\pi}{2n} - \frac{1}{2}s_n\right)}{\nu - \frac{1}{4}s_n}\right\}\left\{q^2 + \overline{2+\alpha}\,\omega^2 + \frac{3}{4}\frac{\beta^2\omega^2\left(s_n - 2\cot\frac{\pi}{2n}\right)}{\nu - \frac{1}{4}s_n}\right\}$$

$$= \left\{\overline{1+\alpha}\,q\omega - \frac{\beta^2\omega^2\left(\frac{3}{4}\cot\frac{\pi}{2n} - \frac{1}{2}s_n\right)}{\nu - \frac{1}{4}s_n}\right\}^2$$

or

$$\alpha q^2(q^2 + \overline{2+\alpha}\,\omega^2) - (1+\alpha)^2 q^2\omega^2$$

$$= \frac{\beta^2\omega^2}{\nu - \frac{1}{4}s_n}\left\{\left[q^2 + 2q\omega(1+\alpha) + (2+\alpha)\omega^2\right]\left[\frac{3}{4}\cot\frac{\pi}{2n} - \frac{1}{2}s_n\right] - \frac{3}{4}\alpha q^2\left(s_n - 2\cot\frac{\pi}{2n}\right)\right\}.$$

But in the terms of order β^2, we can equate the longitudinal and transverse masses, which for either the rigid or the Lorentz electron are equal, to an order β. We can also write $q = \omega$ in such terms, and it is evident that a relative error β^2 is also concerned in this process. For it is now apparent that no correction of order β can appear in the period equation of the present case, which is therefore similar to that of transverse vibrations. Finally, therefore, to order β^3 inclusive,

$$\alpha q^2(q^2 + \overline{2+\alpha}\,\omega^2) - (1+\alpha)^2 q^2\omega^2 = \frac{\beta^2\omega^4}{\nu - \frac{1}{4}s_n}\left\{\frac{15}{2}\cot\frac{\pi}{2n} - \frac{19}{4}s_n\right\}.$$

Specification of the Electron.

We shall consider three electrons of historical interest. In the first case, of the rigid electron of Abraham and others, the well-known expressions for the longitudinal and transverse masses m', m'' are

$$m' = \frac{m_0}{\beta^3}\left\{\frac{2\beta}{1-\beta^2} - \log_e\frac{1+\beta}{1-\beta}\right\},$$

$$m'' = \frac{m_0}{\beta^3}\left\{-2\beta + (1+\beta^2)\log\frac{1+\beta}{1-\beta}\right\},$$

where m_0 is the mass for slow speeds. Expressed as series,

$$\frac{m'}{m_0} = 1 + \frac{6\beta^2}{5} + \frac{9\beta^4}{7} + \ldots$$

$$\frac{m''}{m_0} = 1 + \frac{2}{5}\beta^2 + \frac{9}{35}\beta^4 + \ldots$$

so that

$$\alpha = m'/m'' = 1 + \tfrac{4}{5}\beta^2$$

to order β^3. The corresponding period equation for *a ring of Abraham electrons* is

$$q^2(q^2 - \omega^2) = \frac{\beta^2\omega^4}{\nu - \frac{1}{4}s_n}\left\{\frac{15}{2}\cot\frac{\pi}{2n} - \frac{4\nu}{5} - \frac{91}{20}s_n\right\},$$

where, in the β^2 term, we have written $q = \omega$.

In the case of the Lorentz contracted electron, the masses are given by

$$m' = m_0/(1 - \beta^2)^{\frac{3}{2}},$$

$$m'' = m_0/(1 - \beta^2)^{\frac{1}{2}},$$

so that

$$\alpha = m'/m'' = 1/1 - \beta^2 = 1 + \beta^2,$$

leading to a period equation

$$q^2(q^2 - \omega^2) = \frac{\beta^2\omega^4}{\nu - \frac{1}{4}s_n}\left\{\frac{15}{2}\cot\frac{\pi}{2n} - \nu - \frac{15}{4}s_n\right\}$$

for the Lorentz electron ring.

In the case of the contracted electron which preserves a constant volume, used by Bucherer* and Langevin,† the masses become

$$\frac{m'}{m_0} = (1 - \beta^2)^{-\frac{1}{3}}(1 - \tfrac{1}{3}\beta^2) = 1 + \beta^2 + \ldots$$

$$\frac{m''}{m_0} = (1 - \beta^2)^{-\frac{1}{3}} = 1 + \tfrac{1}{3}\beta^2 + \ldots$$

or

$$\alpha = 1 + \tfrac{2}{3}\beta^2 + \ldots$$

and the period equation of *a ring of constant volume electrons* is

$$q^2(q^2 - \omega^2) = \frac{\beta^2\omega^4}{\nu - \frac{1}{4}s_n}\left\{\frac{15}{2}\cot\frac{\pi}{2n} - \frac{2}{3}\nu - \frac{47}{12}s_n\right\}.$$

It is not necessary to recapitulate the usual arguments as to the nature of the electron. Lorentz has given‡ an apparently fatal argument against the constant volume electron, and that of Abraham has been discredited in various ways. The experiments of Bucherer appear§ to show that the Lorentz mass formulæ are effectively correct, and we shall accordingly adopt this electron for detailed calculation, noticing, however, that the difference between the three electrons, in the period equation, is almost negligible in any case.

* *Mathematische Einführung in die Elektronentheone*, Leipzig, 1904, p. 57.

† "La physique des électrons," *Revue générale des sciences*, 16 (1905), p. 257.

‡ *Theory of Electrons*, English translation, p. 220.

§ *Phys. Zeit.*, 9 (1908), p. 755; *Ber. d. deut. Phys. Ges.*, 6 (1908), p. 688.

Application to Nebulium.

The second approximation to q/ω becomes

$$q/\omega = 1 + \frac{\beta^2}{\nu - \frac{1}{4}s_n}\left\{\frac{15}{4}\cot\frac{\pi}{2n} - \frac{\nu}{2} - \frac{15}{8}s_n\right\}.$$

For nebulium when neutral,

$$\beta^2 = 0^{4}1849, \quad \nu - \tfrac{1}{4}s_n = 3{\cdot}043, \quad \cot\frac{\pi}{2n} = 2{\cdot}414, \quad s_n = 3{\cdot}83$$

$$\frac{15}{4}\cot\frac{\pi}{2n} - \frac{\nu}{2} - \frac{15}{8}s_n = -0{\cdot}129,$$

and

$$q/\omega = 0{\cdot}99999922$$

is the value correctly to eight places. The correction is absolutely inappreciable, even to hundredths of a tenth metre.

We can now deduce the value of m/M for a nebulium atom very accurately. For the ratio of the two principal lines now becomes

$$\{\nu/(\nu - \tfrac{1}{4}s_n + {\cdot}00002)\}^{\frac{1}{2}}\left(1 + 2\frac{m}{\mathrm{M}}\right),$$

where $\nu = 4$, $\nu - \frac{1}{4}s_n = 4 - \frac{1}{4}(1 + 2\sqrt{2}) = 3{\cdot}04289$. The corresponding lines in the nebular spectrum, measured accurately by Wright (the accuracy being tested by the hydrogen lines) are

$$\lambda = 5006{\cdot}89, \quad 4363{\cdot}37,$$

and

$$\left(\frac{4}{3{\cdot}04291}\right)^{\frac{1}{2}}\left(1 + 2\frac{m}{\mathrm{M}}\right) = \frac{5006{\cdot}89}{4363{\cdot}37},$$

which leads to

$$\frac{m}{\mathrm{M}} = {\cdot}000415,$$

where only the third figure is approximate, if Wright's wave-lengths are correct. If we can regard the experimental weight H of the hydrogen atom as correctly given by

$$\frac{m}{\mathrm{H}} = {\cdot}000545$$

(the mean of the best estimates), the atomic weight of nebulium becomes

$$\frac{\mathrm{M}}{\mathrm{H}} = 1{\cdot}31$$

with a possible error of unity in the last figure.

It is unfortunate that the wave-lengths in the coronal spectrum cannot be found with similar accuracy, so as to allow a more precise deduction of the atomic weights of the substances concerned.

Planetary Motion in Space-Time of any Constant Curvature according to the generalised Principle of Relativity. By L. Silberstein, Ph.D., Lecturer in Natural Philosophy at the University of Rome.

Starting from the modification of Einstein's new theory recently proposed by de Sitter,* *i.e.* abolishing Einstein's hypothetical "world-matter," and assuming the four-dimensional world to be of constant curvature without, however, prejudicing its *sign* or amount, I find for the motion of a planet round its sun the following results:—

Let the differential invariant G, proportional to the world-curvature, have throughout the world, outside of matter, the constant value G_0, positive, nil or negative. Write $G_0 = \frac{12}{R^2}$, so that R, a length, may be real, finite or infinite, or imaginary. Further, let M be the rest-mass of the Sun in astronomical units, and

$$\alpha = \frac{M}{c^2},$$

a length which for our Sun, for instance, is about 1·5 km. Finally, let r, ψ, θ be the polar co-ordinates of the planet, with Sun as origin. Then, assuming α/r to be a small fraction, I find for the line-element ds, as a sufficiently accurate solution of the amplified field equations,

$$ds^2 = \left(1 - \frac{2\alpha}{R}\cot\frac{r}{R}\right)\cos^2\frac{r}{R}\,.\,dl^2 - \left(1 + \frac{2\alpha}{R}\cot\frac{r}{R}\right)dr^2 - R^2\sin^2\frac{r}{R}\,.\left(d\psi^2 + \sin^2\psi\, d\theta^2\right) \quad . \quad . \quad . \quad (1)$$

where $l - ct$. For a homaloidal or Euclidean world ($R = \infty$) the element (1) reduces to Einstein's well-known solution of 1916.† For an elliptic world ($R^2 > 0$) the argument of the trigonometrical functions is real, and $\frac{1}{R}\cot\frac{r}{R}$ is, by the way, the elliptical equivalent of the Newtonian potential $\frac{1}{r}$, viz. a solution of $\Delta^2\phi = 0$ in elliptic space; in absence of gravitation, *i.e.* for $\alpha = 0$, (1) is a rigorous solution of the field equations, coinciding with de Sitter's (8^B), *loc. cit.* Finally, if the world‡ is hyperbolic—that is, if $R^2 < 0$—put $R = iR'$; then R′ will take the place of R, and cot, cos², sin² will be replaced

* *M.N.*, 78, 3.

† Cf. *Ann. d. Physik.* 49, 819, where a stands, through an obvious misprint, for $2a$.

‡ To avoid undesirable confusion, the term "world" is strictly used throughout for *the* four-dimensional manifold, and "space" for *a* three-dimensional space.

by the corresponding hyperbolic functions of r/R', without change of sign of any of the terms. Thus (1), and its consequences, need not be written separately for the three sub-cases.

The equations of motion of the planet, $\delta\int ds = 0$, follow at once from the above expression of the line-element. Here it will be enough to give the differential equation of the orbit itself. This is, with $\psi = \text{constant} = \frac{\pi}{2}$, and with the rejection of unimportant terms,

$$\frac{d^2\rho}{d\theta^2} + \rho = \frac{a}{\mathrm{R}k^2}\left[1 + 3k^2\rho^2 - \frac{\mathrm{R}}{a\rho^3}\right] \quad . \quad . \quad . \quad (2)$$

where $\rho = \frac{\mathrm{R}}{r}$ and k a constant (a first integral of the equations of motion). Whence, treating the second and the third terms in the brackets as small perturbational terms, the orbit is an ellipse,

$$\rho = \frac{a\mathrm{R}}{b^2}\left[1 + e\cos(\theta - \omega)\right], \quad k \doteq \frac{2\pi ab}{\mathrm{R}c\mathrm{T}}, \quad a \doteq \frac{4\pi^2 a^3}{c^2\mathrm{T}^2},$$

with slowly variable elements. While secular changes of a, e (just as for $\mathrm{R} = \infty$) are absent, the secular motion of the perihelion consists of two parts, due to the two supplementary terms in (2), namely (rejecting in the second part the sixth power of eccentricity), per period of revolution T,

$$\epsilon = \epsilon_1 + \epsilon_2 = \frac{6\pi}{1 - e^2}\left(\frac{2\pi a}{c\mathrm{T}}\right)^2 + \frac{3\pi}{4}\left(1 - \frac{4}{3}e^2 + \frac{5}{8}e^4\right)\left(\frac{c\mathrm{T}}{\pi\mathrm{R}}\right)^2 \quad . \quad (3)$$

The first part, ϵ_1, is the well-known expression, giving $43''{\cdot}1$ per century for Mercury; it is independent of the curvature and identical with Einstein's original result for a homaloidal world. In other words, if $\mathrm{R} = \infty$ then ϵ_1 represents the whole motion of the perihelion. The new part of the motion, ϵ_2, is essentially due to curvature, and would vanish in a homaloidal world. It is positive or progressive for an elliptic world, negative or retrograde for a hyperbolic one, and proportional to the squared ratio of the light path $c\mathrm{T}$ in one (planetary) year to the radius of the world curvature, while ϵ_1 is proportional to the squared ratio of the length of the orbit to that light path. These relations seem particularly interesting. A point of minor importance is that, while ϵ_1 increases, ϵ_2 decreases with increasing eccentricity. For nearly circular orbits the ratio of the two motions of the perihelion becomes

$$\epsilon_2 : \epsilon_1 = \tfrac{1}{8}\left(\frac{c\mathrm{T}}{2\pi a} \cdot \frac{c\mathrm{T}}{\pi\mathrm{R}}\right)^2 \quad . \quad . \quad . \quad (3a)$$

The first factor of the squared product is a large number, the second a small fraction, possibly much smaller than the reciprocal of the first.

It is now generally believed that ϵ_1 represents by itself well

enough the chief of the observed excesses of secular motion. If so, then the new motion ϵ_2, due to finite world-curvature, can be but a small fraction of ϵ_1, at least in those cases which have been ascertained. Thus, for example, Mercury's observed excess, 43″ per century, is entirely accounted for by ϵ_1 itself, and it must therefore be assumed that in this case ϵ_2/ϵ_1 is a small fraction, positive or negative. This will give a lower limit for $|R|$. Suppose that, by future improvements of the data and of their discussion, Mercury's observed excess is made so accurate as to enable us to assert that it is represented by ϵ_1 alone within one in a thousand parts; then $|\epsilon_2 : \epsilon_1| < 0{\cdot}001$, and therefore, by (3*a*),

$$|R| > 11{\cdot}2\frac{c^2T^2}{2\pi^2 a},$$

and since for Mercury, in astronomical units, $a = 0{\cdot}387$, $T = 88$, and $c = 174$, we should have

$$|R| > 2{\cdot}10^9.$$

Such, then, would be the lower limit of the radius of world-curvature;* that is to say, if the world is elliptic, the total length of a straight line in natural measure, πR, would exceed $6{\cdot}4 \cdot 10^9$, and if hyperbolic ($R^2 > 0$), that length will be infinite, of course, but the "defect" of a triangle ($180° -$ sum of angles) will be smaller than $\frac{1}{4}10^{-18}$ radians, or $5{\cdot}''10^{-14}$, per unit area in astronomical units. From the displacement of the spectrum lines of the helium stars de Sitter finds (*loc. cit.*, p. 27), as a rough estimate, $R = \frac{2}{3} \cdot 10^{10}$, a little beyond the above limit—the sign of his R^2 being positive.

It would, of course, be more desirable to find for $|R|$ not the lower but the upper limit, or better still, to find its value between certain more or less narrow limits. In our case this would be possible if, for instance, the observed excess ϵ for some planet were decidedly too small or too great to be accounted for by ϵ_1 alone; the remainder or the defect would then be given by ϵ_2, thus enabling us to determine the curvature of the world. If $\epsilon > \epsilon_1$, then we should claim a positive, and if $\epsilon < \epsilon_1$, a negative curvature for our world, notwithstanding that Einstein and others think to have found some reasons (in connection with the behaviour of the fundamental tensor "at infinity") speaking in favour of an elliptic world. No matter how strong such a-prioristic reasons for a positive world-curvature might be, it seems better to leave the decision not only of the amount but also of the sign of the curvature to the future astronomer and astrophysicist. The more so as Einstein's reasons are by no means indisputable.

At any rate, the above formula (3), bringing into evidence the hierarchy of the three lengths

$$2\pi a,\ cT,\ \pi|R|$$

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* In the present state of things it would, of course, be enough to claim $|\epsilon_2 : \epsilon_1| < \frac{1}{100}$, which would give $|R| > \frac{2}{3} \cdot 10^9$.

in their relative importance, has seemed to be of some interest. The fundamental properties of a hyperbolic world, in their contrast to those of Einstein's spherical or elliptic world, will be discussed at a later opportunity.

London; 1918 *January*.

Note, added March 8th.—Einstein's reasons for giving up his originally homaloidal world are these: The tensor components $g_{11} = g_{22} = g_{33} = -1$, $g_{44} = 1$ (others zero) in a field of pure inertia, *i.e.* far away from gravitating matter, "are not invariant"; the boundary values "at infinity" are different in different systems of reference. "Einstein and others," in de Sitter's words, "have therefore tried to find another extrapolation by which the $g_{\mu\nu}$ . . . would at infinity degenerate to a set of values which would be the same for all systems of reference" (*M.N.*, **78**, 4).

As far as I can see, this difficulty is based upon some strange oversight. In fact, to postulate the above tensor as a limiting value amounts to assuming that the line-element "at infinity" should be reducible to the particular form

$$ds^2 = dx_4^2 - dx_1^2 - dx_2^2 - dx_3^2.$$

Now, this requirement can well be expressed *in a general covariant form*, viz. by requiring that all the four-index symbols

$$(lkhi) = \frac{\partial}{\partial x_i}\left[\begin{matrix} lh \\ k \end{matrix}\right] - \frac{\partial}{\partial x_h}\left[\begin{matrix} li \\ k \end{matrix}\right] + \sum_p \left[\left\{\begin{matrix} li \\ p \end{matrix}\right\} \left[\begin{matrix} hk \\ h \end{matrix}\right] - \left\{\begin{matrix} lh \\ p \end{matrix}\right\} \left[\begin{matrix} ik \\ p \end{matrix}\right] \right]$$

should vanish at infinity. These symbols, of which there are $\frac{4^2}{12}(4^2 - 1) = 20$ independent ones, constitute a tensor of the fourth rank, the Riemann-Christoffel tensor (from which Einstein himself has derived his field equations). If they vanish in one reference system, they will vanish in all others.

It is well known that $(lkhi) = 0$ is the necessary and, according to Lipschitz, also the sufficient condition for the line-element of any n-dimensional manifold, $ds^2 = \Sigma g_{ij} dx_i dx_j$, to be reducible to a quadratic form with constant coefficients,—in our case, tending to such a form (*cf.* W. Killing, *Nicht-Euklidische Raumformen*, Leipzig, 1885, p. 230).

Thus there is, it would seem, no real difficulty "at infinity," and consequently no such reason for discarding a priori an infinite, *i.e.* homaloidal or hyperbolic world in favour of an elliptic one. The world-curvature is the synonym of some real, phenomenal contents, and cannot, therefore, be found out by mere reasoning.

The Number of Stars of Different Magnitudes in the Hyderabad Astrographic Catalogue; Second Paper, Zone −18°. By R. J. Pocock, B.A., B.Sc.

1. A discussion of the numbers of stars of different magnitudes in the *Hyderabad Astrographic Catalogue*, vol. i., will be found in *Monthly Notices, R.A.S.*, **77**, 472. As this is the only zone of which counts have as yet been published between Algiers (−1°) and Perth (−32°), it seemed desirable to support the evidence of this single zone by that of another at as early a date as possible. With this end in view the plates in −18° have been counted as they were measured, so that on the completion of the zone early in 1917 December the material was ready for discussion. It will be seen below that the results from this zone present some interesting differences from those for zone −17°.

2. The practice adopted for determining the magnitudes of the stars in vol. i. has been followed in zone −18°, and although some improvement has been effected in the estimates towards the end of the work on this zone, the results are still far from satisfactory. It is hoped soon to make some experiments with the method adopted at Perth of comparing the images with a set placed in the eyepiece of the measuring instrument, a method which has been found to give very good results,* and possibly this method may be extended to all the Hyderabad zones. For the present discussion, however, the estimated diameters have been used, and errors of the scale will not affect the results which do not depend on absolute measures of magnitude but only on relative magnitudes.

3. The work on this zone was divided into several sections, which were taken and measured at different times, the whole extending over nearly three years. The groups were separated by considerable intervals, but within each group the plates were taken within a comparatively short time. The plates from $5^h\ 30^m$ to 9^h were taken first, after an interval of about a year the plates from 9^h to 19^h were taken, some months later about half the plates from 21^h to 0^h, and all the plates from 0^h to $5^h\ 30^m$ were taken; nearly a year later all the plates in 19^h and 20^h and the remaining plates in 21^h to 0^h were taken. There are thus gaps in the series between 5^h and 6^h at 9^h and 19^h. At 21^h and 0^h there was a change but no sharp break, since the previous and succeeding groups are intermingled in this region.

The exposures have varied with the age of the plates, but have been nearly constant within each group, and have been such as to give a good average number of stars throughout the zone.

4. The actual numbers counted were multiplied by factors in order to make them equivalent to ten plates in each hour of R.A. The logarithms of these numbers for N_{50}, N_{30}, N_{20}, N_{12}, N_8 were then tabulated for each hour, the column N_{30} was subtracted from the other columns, and then the mean of each column was sub-

* *Monthly Notices, R.A.S.*, **77**, 618.

tracted from each number in the column; the resulting numbers multiplied by 100 are given in Table I.

The values of log N_{30} ($\times 100$) are given in the second column, and the mean of each column is given at the bottom; these enable the original numbers to be recovered if desired.

TABLE I.

Ratios of log N to log N_{30} for selected Magnitudes in Units of 0·01.

R.A. h	N_{30}	N_{50}	N_{20}	N_{12}	N_8	Gal. Lat.
0	263	− 3	0	− 1	− 10	− 78
1	259	+ 10	0	+ 2	− 6	− 76
2	270	− 5	+ 1	− 4	− 10	− 64
3	274	− 15	− 3	− 7	− 12	− 51
4	286	0	− 2	− 7	− 12	− 38
5	279	+ 14	− 3	0	0	− 24
6	289	− 3	+ 2	+ 3	+ 5	− 11
7	290	+ 3	+ 10	+ 20	+ 27	+ 1
8	259	+ 1	+ 8	+ 15	+ 30	+ 13
9	264	− 15	+ 2	+ 13	+ 23	+ 24
10	282	− 13	− 4	− 3	+ 2	+ 34
11	271	− 4	− 5	− 10	− 8	+ 41
12	268	− 3	− 4	− 5	+ 3	+ 44
13	269	− 4	− 9	− 12	0	+ 43
14	269	− 15	− 3	− 7	− 3	+ 38
15	291	+ 13	− 10	− 18	− 18	+ 30
16	268	− 1	− 4	− 10	− 9	+ 19
17	287	0	+ 3	+ 3	+ 15	+ 7
18	310	+ 14	+ 6	+ 9	+ 13	− 4
19	263	+ 4	+ 7	+ 17	+ 15	− 17
20	275	+ 13	+ 4	+ 4	− 12	− 30
21	280	+ 13	+ 3	0	− 11	− 43
22	269	− 2	0	0	− 11	− 57
23	257	+ 2	+ 1	− 4	11	− 70
Mean.	...	75	28	52	73	...

5. With regard to the actual numbers of stars in different parts of the zone, there is a sharp rise between 6^h and 7^h, but not to quite the same extent as in $-17°$; on the other hand, the rise from 16 to 18 is very much greater than in $-17°$, with a very sudden drop between 18^h and 19^h. The total number of stars measured

in this zone, viz. 61,379, is slightly less than in the previous zone, and gives an average of 340 stars per plate.

6. In Table II. are shown the magnitudes corresponding to the columns of Table I., determined on the C. and M. scale from the table in *Monthly Notices*, **75**, 145, together with the magnitudes computed from the formula $M = 15{\cdot}44 - 0{\cdot}88\sqrt{d}$. This formula was determined independently for this zone, but is the same as that determined for the zone -17° in the previous paper. The habits of the measurers do not appear to have changed much during the three years they have been at work on these two zones.

The differences between the calculated and observed magnitudes for zone -17° are also given for comparison, and it will be seen that the errors of the scale are practically the same for both zones.

TABLE II.

Comparison of calculated and observed Magnitudes.

Diameter	51	30	20	12	8
C. and M. Mag.	9·12	10·84	11·55	12·22	12·90
Calculated	9·16	10·62	11·51	12·40	12·95
Difference	−·04	+·22	+·04	−·18	−·05
Difference, Zone -17°	−·02	+·15	+·04	−·15	−·01

7. Considering Table I. it will be seen that the material is obviously divisible into four groups, at 5^h, 9^h, 16^h, and 19^h.

The divisions at 5^h and 19^h correspond closely to those at 6^h and 20^h in zone -17°, being moved back one hour in each case; as a matter of fact, the division at 19^h might be moved one hour forward or that at 20^h in -17° one hour back. The division at 5^h coincides with one of the breaks in the work mentioned in 3, and if it stood alone we might suspect it of being artificial, but it is supported by the evidence from -17°, where there is no such suspicion, and there seems no sufficient reason for not accepting it as real. The division at 19^h is not only supported by the previous zone, but it occurs at a place where no sharp division is to be expected.

The breaks at 9^h and 16^h, however, did not appear in zone -17°, and that at 9^h coincides with a gap in the work; but we may note (i.) the break at 16^h does not occur at a gap, and if the break at 16^h is real, there must be another somewhere between 5^h and 16^h, and it may be mere chance that it happens to be at 9^h. (ii.) Although in -17° there is no break at 9^h and 16^h, the region 12^h to 16^h gives negative or very small positive log ratios, and probably corresponds to the region 10^h to 16^h in this zone, its character being masked by accidental errors.

We cannot say definitely in the present case that none of the discontinuities are introduced by artificial conditions as we could say in discussing -17°, at the same time there is strong reason for accepting them as real.

8. In what follows we shall designate the four regions of Table I. as follows:—

		h		h	
Region	I.	20	to	5	"Obscured."
,,	II.	6	,,	9	"Unobscured."
,,	III.	10	,,	16	"Obscured"
,,	IV.	17	,,	19	"Unobscured."

We may note before proceeding further that the recognition of region III. as an obscured region is not at variance with results previously obtained by Professor Turner. The Algiers' results, which are the nearest to Hyderabad on the northern side, gave an obscured region from 10^h to 18^h, and looking at Professor Turner's diagram in *Monthly Notices*, **75**, 480, we see that it is only necessary to continue the right-hand "obscured patch" a little further south to include region III. of this paper. When the diagram was published there was no evidence between $-1°$ and $-32°$, and the exact position of the southern boundary of this patch was therefore quite uncertain. The region I. can also be fitted into the diagram quite consistently with the results from other zones.

One other point about Table I. calls for attention. It will be seen from the last column that groups II. and IV. are close to the galaxy, while the obscured patches I. and III. are situated in high galactic latitudes. It appeared in the previous paper that the ratio of faint stars to bright in the Hyderabad zones bore little relation to the galaxy. The recognition of region III., as an obscured patch, however, shows that this is not so, and that in the Hyderabad zones the galactic condensation is mixed up with the obscuration in such a way that it is difficult to separate the two.

9. Before deducing the amount of the obscuration the actual counts were divided or multiplied by factors so as to make them equivalent to about 2000 stars in each hour of R.A. What follows depends on these modified counts. The amount of the obscuration is shown in Table III., which is similar to Table III. of the former paper.

TABLE III.

Amount of Obscuration, Zone $-18°$. *Logarithms in Units of* 0·01.

Mag.	9·12.	10·84.	11·55.	12·22.	12·90.
I.	800	4612	8746	14532	20035
II.	187	949	2000	4096	8193
III.	545	2911	4809	7671	13964
IV.	137	807	1714	3320	6084
log I.	0	76	104	126	140
log II.	0	71	103	134	164
log III.	0	72	94	114	140
log IV.	0	77	109	138	164
log (I. + III.)/(II. + IV).	0	0	−14	−32	−48
Result for $-17°$	0	+4	−8	−17	−42

10. The results for $-17°$ have been shown for comparison. The amount of the obscuration comes out rather larger in $-18°$, but it must be remembered that region III., which is now treated as obscured, was treated as unobscured in the first paper. The final number -48 is a little larger than is found in other zones, but of the same order of magnitude. The result previously found, that in the Hyderabad zones the obscuration begins about 11·0, is confirmed. The numbers fit in well with adjacent zones; if we make the final number -42, the comparison becomes:

Mag.	10·0.	11·5.	13·0.
Algiers $-1°$	-10	-18	-42
Hyderabad $-17°$	0	-7	-42
,, $-18°$	0	-7	-42
Perth $-32°$	0	0	-42

11. The amount of galactic condensation was deduced in the previous paper from results shown in Table IV. of that paper. At the foot of the seventh column of that table $\pm 70°$ is a misprint for $\pm 7°$. There also appears to have been an error in the calculation of the coefficient for the obscured region, which now comes out $-·00126$ instead of $-·00142$, and the result now falls between those for Algiers and Perth. But had the region 12^h to 16^h been treated as obscured the results would have been different, and too much stress must not be laid on the agreement of these coefficients, which at the best are rather crudely calculated.

In Table IV. below, the results of Table I. are arranged according to galactic latitude.

TABLE IV.

Amount of Galactic Condensation.

Obscured region.						Unobscured region.					
Mean Gal. Lat.	No. in Group.	9·12.	11·55.	12·22.	12·90.	Mean Gal. Lat.	No. in Group.	9·12.	11·55.	12·22.	12·90.
$\pm 58°$	9	-1	-1	-3	-8	$\pm 16°$	4	-3	$+5$	$+12$	$+18$
$\pm 32°$	8	$+1$	-3	-6	-8	$\pm 4°$	3	$+4$	$+5$	$+8$	$+14$

12. It has already been remarked that the galactic condensation is mixed up with the obscuration in this zone in such a way that their separation is difficult. For the obscured region the range of latitude is fairly large, and there is clearly no great condensation, rather the reverse, while for the unobscured region the range of latitude is too small to tell us much; the fact is that in separating the obscured and unobscured regions we have also separated the galactic and non-galactic regions. The actual coefficients per magnitude per degree worked out from the above table give the results

Coefficients of Galactic Condensation.

Unobscured region $+·00015$. Obscured region $+·00037$.

The former result agrees fairly well with the Perth result, which gives a very small coefficient for the unobscured region. The Cape zone -41° gives a fairly low result for the obscured region, but Perth and Cape -42° give a much higher coefficient. It is probable that the above results are of little value.

13. As an alternative we may determine the galactic condensation without taking any account of the obscured patch; we then find the results shown in Table V.

TABLE V.

Amount of Galactic Condensation for the whole Zone.

Mean Gal. Lat.	No. in Group.	9·12.	11·55.	12·22.	12·90.
$\pm 50^\circ$	12	−2	−2	−4	−9
$\pm 18^\circ$	12	+3	+2	+4	+8

and from these the coefficient per magnitude per degree comes out

$$\text{Coefficient of galactic condensation} = -\cdot00176.$$

This is quite a reasonable coefficient, rather larger than those obtained from the northern zones, but it is impossible to say whether it lends support to the hypothesis that the galactic condensation is greater for the southern hemisphere, since we cannot say how much of this is really an effect of the galaxy; if the results of the previous paragraph have any value, very little of it is a true galactic effect, most of it being due to the effect of the obscured spiral.

14. Writing $\alpha + 3\cdot66\delta = 247^\circ$ and substituting $\delta = -18^\circ$ we get $\alpha = 313^\circ$, so that the minimum should be about 21^h. As a matter of fact the figures under the heading N_8 in Table I. are remarkably uniform within each group, and there is little indication of any minimum within the group I. The discontinuity at -19^h is much sharper than that at 5^h for the fainter stars (for the brighter stars the discontinuity occurs considerably later); this is in accordance with the results found by Professor Turner (*Monthly Notices, R.A.S.*, **76**, 153). The numbers given in Table II. of the former paper suggest that there is little difference between the discontinuities at 6^h and 20^h, and if the numbers for N_8 in that table are added to the numbers for N_8 in Table I. above, we find that although the difference is not great there is a rather sharper break at 19^h than at 6^h; if, as Professor Turner supposes, the sharper edge of the spiral changes sides between north and south, the change probably takes place near the Hyderabad zones, which would explain the fact that there is not much difference between the two edges of the spiral in the Hyderabad zones. Since such difference as there is is in the same direction as found for the Cape zones, the crossing over must occur a little further north than the Hyderabad zones.

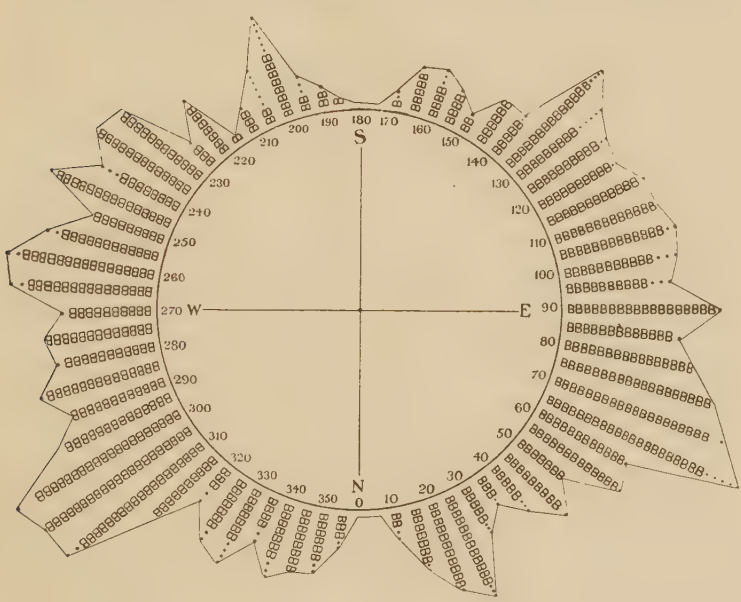
Nizamiah Observatory, Hyderabad:
1917 *December* 5.

Solar Prominences, 1917. By G. J. Newbegin.

The year 1917 has permitted seventy-eight observations to be made, resulting in a total of 835 prominences being observed and recorded, thus making a mean daily average of 10·7.

From the above total of all prominences a deduction of 227 is necessary before making a diagram in accordance with the scheme of the International Solar Union. This deduction leaves 608 prominences with which to build the diagram now presented.

The number of position angles affected amounts to 760; it will be noticed that the heliographic poles show slight vacancies.



B = bright. · = faint.

Distribution of Prominences, 1917.

The diagram indicates that the balance of activity is considerably in favour of the northern hemisphere, while west and east differ but little. Prominences on the equatorial line have been apportioned equally to north and south, the figures thus being:

North, 422	South, 339	=761.
East, 384	West, 377	=761.

The 608 prominences from which the diagram is built up give an average height of 44″·7.

Dark absorption was noticed on forty-three occasions; 39 on bright and 4 on faint prominences, sometimes at considerable elevation.

Observations of Encke's Comet (1917 *c*) *made with the 28-inch Equatorial at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

	G.M.T.	R.A. Comet−*.	R.A. Corr. for Refr.	R.A. Log Factor of Parallax.	Decl. Comet−*.	Decl. Corr. for Refr.	Decl. Log Factor of Parallax.	Star.
	1918 d h m s	s	s		"	"		
Mar.	1 7 27 47·74	+61·11	−·03	9·5682	...	...	...	*a*
	7 37 28·94	...	...	...	−132·3	−·7	0·8248	*a*
	7 47 38·37	−78·55	−·15	9·5702	...	...	...	*b*

Places of Comparison Stars.

Star.	Name.	Mag.	R.A. 1918·0.	Decl. 1918·0.	Reduction to Apparent Place.		Authority
	°		h m s	° ′ ″	s	″	
a	B.D.+10·65	7·3	0 33 2·19	N. 10 59 7·68	+0·51	+4·67	Leipzig A.G. Cat.
b	B.D.+10·70	7·3	0 35 23·80	11 4 55·89	+0·52	+4·71	,, ,,

The comparisons of R.A. were made by taking transits over a pair of wires in the north and south direction, those of Declination by successive bisections of star and comet by a micrometer wire at right angles to the former. There were 12 comparisons with star *a* in R.A., 20 in Declination, and 4 with star *b* in R.A.

The comet appeared as a circular diffused mass, about 2′ in diameter, with no definite condensation or nucleus. Measures were therefore made with difficulty, the estimated centre of the nebulosity being observed.

Taking the value of log Δ as 0·0920, and reducing the observations to G.M.T. $7^h\ 37^m\ 28^s{\cdot}94$, the rate of motion in R.A. being taken as $+0^s{\cdot}57$ in 6 minutes, the apparent place of the comet (corrected for parallax) at this time is

		h m s	° ′ ″	
From star	*a*	R.A. 0 34 4·98	N. Decl. 10 57 4·77	
,,	*b*	R.A. 0 34 4·95		

Assuming that Mr. Viljev's elements (see *M.N.*, **78**, 292) are all correct except T, the time of perihelion passage, the value of T deduced from these observations is 1918 March 24·295 G.M.T., which is $0^d{\cdot}018$ less than Viljev's value. The small residual, and the fact that R.A. and Declination give nearly the same result, indicate that all his elements are close to the truth.

The observations were made by Mr. R. Jonckheere.

Royal Observatory, Greenwich :
March 8, 1918.

Preliminary Values of Variation of Latitude at Greenwich during 1917.

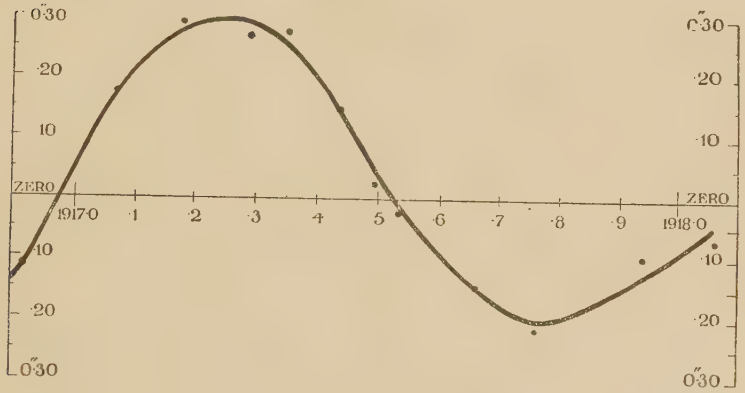
(*Communicated by the Astronomer Royal.*)

In continuance of results published in *M.N.*, **76**, 739, and **77**, 443, preliminary values of latitude variation at Greenwich in 1917 are here given.

These have been deduced from the measurement of 201 plates taken with the Cookson Floating Zenith Telescope during the period.

For details of reduction, etc., reference is directed to Mr. Jones' paper, *M.N.*, **75**, 42.

The quantities under the heading "Variation" are Observed *minus* Mean Latitude.



Latitude Variation at Greenwich, 1917.

A table of corrections applicable to observed N.P.D.'s for variation of latitude on selected dates is also included.

These were read from a smooth curve drawn through the observed values plotted to scale on a diagram.

Latitude Variation at Greenwich.

Date.	Variation.	P.-E.
	″	″
1916·92	+ ·110	± ·014
1917·07	− ·179	± ·015
·18	− ·291	± ·014
·29	− ·267	± ·013
·35	− ·276	± ·013
·44	− ·149	± ·012
·50	− ·025	± ·020
·54	+ ·023	± ·013
·66	+ ·143	± ·013
·76	+ ·214	± ·012
·94	+ ·098	± ·013
1918·06	+ ·067	± ·012

Corrections applicable to observed N.P.D.'s
(*Longitude of Greenwich.*)

1917 Jan. 1	+″·02	May 1	+″·27	Sept. 1	−″·15
11	+·08	11	+·25	11	−·17
21	+·14	21	+·22	21	−·18
Feb. 1	+·19	June 1	+·18	Oct. 1	−·19
11	+·22	11	+·14	11	−·20
21	+·25	21	+·09	21	−·19
Mar. 1	+·27	July 1	+·04	Nov. 1	−·18
11	+·28	11	+·01	11	−·17
21	+·29	21	−·03	21	−·15
Apr. 1	+·29	Aug. 1	−·07	Dec. 1	−·14
11	+·29	11	−·10	11	−·12
21	+·28	21	−·12	21	−·10
				31	−·08

Errata in Van Biesbroeck's Third Series of Measures of Double Stars. By Eric Doolittle.

This very complete and thorough series of measures comprises 184 pages of vol. xiii. of the *Annals* of the Royal Observatory of Belgium. It is divided into two parts, of which the first contains measures on close pairs and pairs in rapid motion, while the second contains an exhaustive study of all of the *H* pairs which lie between 50° of North Declination and the North Pole. All of the latter (numbering 457 pairs) were measured on at least three nights each, with the exception of three only, which lie too near the Pole to permit of accurate measurement; and afterwards the place of each pair was accurately determined, either from a catalogue of precision, by measurement from a neighbouring star, or from the material of the Astrographic Catalogue. For 166 pairs direct micrometrical comparisons at the telescope were necessary, and in these cases the Right Ascensions were obtained to hundredths and the Declinations to tenths of seconds. This work will thus afford a certain foundation for the determination of the character of any motions in these faint pairs which may in the future be found to exist.

In the process of copying all of this material the following errata were met with:—

PAGE

357. Hough 212. The magnitudes given belong to Burnham 490; the stars of the close pair are of nearly equal brightness; other observers reverse the quadrant at this time.

429. H 1118. *For* **1293** *read* **1292.**

PAGE

432.*1130. A pair answering H's description was measured by Storey (*M.N.*, **70**, 467), but only the rough position was given.

435. H 2190. Measured by Burnham in 1909, who finds a distance 2″·3 greater (*Ast. Nach.*, 4426).

438.*H 2219. Both from the identification (p. 438) and from the measures (p. 519) the R.A. is 1 min. 8 sec. too small, and the Decl. is 2′·6 too small.

442. H 2255. In the comparison on p. 520 −0′ 40″·7 *should be* +0′ 40″·7.

443.*H 2262. The comparison (p. 520) in R.A. is correct; the R.A. given on p. 443 is 1 min. too great. The Decl. is, however, correct, there being an error of sign on p. 520.

445. H 2284. In the Decl., *for* 77° *read* 73°.

446. Anon. This is H 2307. Measured by Burnham in 1907 (*Ast. Nach.*, 4261).

449. H 2354. Measured by Espin in 1911 (*M.N.*, **72**, 193). The H star is 2 min. preceding the star measured by Biesbroeck, which last seems to be new.

450. H 2366. *For* **3495** *read* **3945**. The magnitude of *B* in the measure of *BC* should be 11·4.

451.*H 2382. The comparison on p. 520 seems in error. There is no B.D. star in the place there indicated. The identification and place on p. 451 are correct.

*H 2384. From the comparison (p. 520) the Decl. is 4′·2 too large. It is the place on p. 451 which is correct.

454.*H 2431. The R.A. is 1 min. too great.

456. H 2460. In H's measure the distance and angle are interchanged.

457. H 2492. This is not the H pair. The latter is about 1 min. preceding, and is a brighter and interesting object composed of a 10·8 magnitude star attended by a faint nebula. Measured by Burnham (*Ast. Nach.*, 4427).

459. H 3315. In the mean, *for* 10″·3 *read* 10″·03.

460. H 2523. Called **5307** on p. 521.

475. H 2652. The star here measured is 2 min. 15 sec. following and 32′ north of H's place, and the descriptions also disagree. It seems very probable, therefore, that this is some other pair.

479. H 2846. In the comparison (p. 521) A. G. Hels. 10045 *should be* 10041.

480. H 2848. In H's measure, *for* 30″ *read* 3″.

481. H 1471. *For* 15″·24 *read* 13″·24.

483. H 2913. H's measure belongs with greater probability to *AC*.

486. H 1516. Measured by Burnham in 1907 (*Ast. Nach.*, 4209).

487.*H 1545. If the comparison (p. 522) is correct, the R.A. is 9 sec. too large. The place on p. 487 is correct; there is an error of sign on p. 522.

489. H 1659. Measured by Aitken in 1904 (*Lick Obs. Publ.*, 12).

H 1570. *For* **14066** *read* **10466**.

PAGE

490. H 1588. In the comparison (p. 522) $-0'\ 47''{\cdot}1$ *should be* $+0'\ 47''{\cdot}1$. The pair measured by Burnham in 1910 (*Ast. Nach.*, 4464).

491. H 1621. Measured by Burnham in 1907 (*Ast. Nach.*, 4261).

492. H 1622. Measured in 1907 by Burnham, who adds a nearer companion (*Ast. Nach.*, 4261).

493. H 3033, *AC.* *For* 1901·53 *read* 1910·53.

495. H 1677. This is also Krueger 54, **11132**. There are several measures. An error of 2 min. in the R.A. in the *G.C.*

496. H 3055. In the comparison (p. 522) $+32^{s}{\cdot}32$ *should be* $-32^{s}{\cdot}32$.

Anon. In the comparison (p. 522) $-33^{s}{\cdot}02$ *should be* $+33^{s}{\cdot}02$.

498. H 1710. Measured by Espin in 1909 (*M.N.*, **70**, 238).

500.*H 1751. *AC* is Aitken 1461. Measured by A. in 1906 (*Lick Obs. Bull.*, 109). Aitken identifies this with B.D. 55° 2722; this is the correct identification.

506. H 1830. Measured by Espin in 1908 and two other components added; one at $5''{\cdot}85$ is Espin 691 (*M.N.*, **69**, 225).

507. H 3157. Measured by Burnham in 1910 (*Ast. Nach.*, 4534). The cluster is *h* 2196.

H 1843. In H's second magnitude, *for* 10 *read* 11.

508.*H 1847. The comparison (p. 523) is in error. The R.A. of p. 508 is 16 sec. too small, and the Decl. is 12′ too large.

509. H 1865. *For* 213°·7 *read* 213°·6.

H 1868. *For* A. G. Hels. 12946 *read* 13946.

511. H 1878. In the comparison (p. 523) $+2'$ *should be* $-2'$. This was measured by Espin in 1907 (*M.N.*, **68**, 204).

H 1881. Measured by Burnham in 1905.

513. H 1901. This is evidently H 1899. Measured by Burnham in 1905, but it is probable that the two pairs are identical.

*H 1905. The R.A. is 52·3 sec. and the Decl. is 6′·6 in error.

H 1902. *For* 9m *read* 9·5 mag.

514. H 1904. This pair is DM 59° (2759), 37·4 sec. following and $9''{\cdot}1$ north of H 1902. A closer component was found by Fox, = Fox 50. (*Ann. Dearborn Obs.*, **1**.)

Anon. *For* Decl. 61° *read* 62°.

517. H 1923. Measured by Espin in 1907 (*M.N.*, **68**, 204), and by Burnham in 1910 (*Ast. Nach.*, 4464). Burnham obtains a distance 1″ larger.

518.*H 1930. The comparison star (p. 523) cannot be A.G. Hels. 14417. If we assume that it is 14673, the resulting position agrees practically with the B.D. position. This assumption is correct.

524. G. C. **7318** *should be* **7316**.

Additional Note.

After this list was prepared it was submitted to Dr. Van Biesbroeck, who kindly looked up the sources of error, and when possible, verified the corrections here made. In all cases above marked with a (*) the uncertainties have been definitely cleared up by him, except in the two following:—

PAGE

432. H 1130. "I cannot find any star answering Storey's description on the Greenwich Section of the Astrographic map, yet the pair that I have measured is very plain" (Van Biesbroeck).

475. H 2652. "I do not see how the wrong star could have been observed, because from the map the real one is in the indicated place" (Van Biesbroeck).

The Flower Observatory:
1918 *January* 22.

On the Determination of Photographic Magnitudes. (Second Paper.) By J. Halm.

§ 1. *General Outline of the Problem.*

The object of astronomical photometry is the establishment of the apparent light intensities, I, of the stars with reference to an adopted fixed intensity, I_0, the media through which the impressions of the luminosities are received being the retina of the eye in the case of *visual* observations, and the sensitive film of the plate in the case of *photographic* observations.

Since these media are sensitive to incident light within certain limits of wave-length, the observed light-ratios refer to those integral portions of the radiative energies only which are enclosed within the range of the "effective wave-lengths" of the medium.

Instead of in light-ratios, the brightnesses are usually expressed in terms of "magnitudes," m, the two quantities being related by the equation

$$-2{\cdot}5 \log I/I_0 = m - m_0 \quad . \quad . \quad . \quad (1)$$

In the case of photographic observations the recorded phenomena are represented by certain actinic effects impressed upon the plate.

If the plate is exposed in the focus of the collecting lens, and not too far from its central line of collimation, the images of the stars appear in the form of circular discs whose diameters vary with the brightness of the object. In this case, therefore, the diameters, D, may serve as an empirical measure of the actinic effects. [This method is known, however, to fail in the case of very faint stars.]

If, on the other hand, in accordance with Schwarzschild's method, the plate is exposed at some distance from the focus of the collecting lens, the images appear as circular discs, the *opacity* of which decreases sensibly with the magnitude. The "blackness" of the star images may in this case serve as a measure of the brightness.

The object of astronomical actinometry is to establish the functional relation between the observed diameters (or opacities) and the corresponding magnitudes.

In general the following three quantities enter into the problem, and have to be examined in regard to their interdependence, viz.—

(1) the incident intensity, I;
(2) the time, t, during which the plate is exposed;
(3) the actinic effect, A, expressed as a function of the sizes or opacities of the star images.

Observations can be arranged in such a manner that one of these three quantities is kept constant during the experiment.

(*a*) If I is constant, *i.e.* if the same star is exposed during varying intervals of time, a relation between A and t is obtained, which will be denoted as the "time-scale."

(*b*) If t is constant, *i.e.* if stars of *different* intensities are exposed on the same plate during the same interval of time, a relation between A and I is obtained, which will be referred to as the "intensity-scale."

(*c*) If the actinic effect, A, is kept constant, *i.e.* if different intensities are exposed during such intervals of time that equal images are formed, a relation between I and t is obtained which may be called the *isoactinic*.

According to the definitions, the ultimate object of astronomical actinometry is the establishment of the *intensity-scale* for a given plate, from which the intensity I, or the magnitude m, of any star exposed during a stated time-interval may be obtained by means of its actinic effect expressed as a function either of the observed size or of the opacity of its image.

The time-scale and intensity-scale must under all circumstances fulfil the condition that the times and intensities associated with the *same* actinic effect conform with the properties of the isoactinic. If, therefore, the time-scale and the isoactinic of a given plate are known, the desired intensity-scale can be found. The problem thus resolves itself into establishing the relationship between A and t for a constant value of I on the one hand, and the relationship between I and t for constant A on the other.

It will be noted that this method is based on comparisons of *equal* actinic effects under two different conditions. Hence the question whether these effects are represented as functions of the sizes or opacities is irrelevant. The only essential point to be observed in measuring the actinic effects is the attainment of the highest possible accuracy in the estimation of their *equality*.

There is obviously no difficulty in finding the time-scale

directly from observations. It is only necessary for this purpose to expose the plate successively during varying intervals of time. Any star will then give the required relation between A and t, where A is a function of the measured quantities D (diameters or opacities). The time-scale can then be represented in the form of a curve, the adopted co-ordinates being D and t, or more conveniently log t.

To establish directly the isoactinic for the plate in question is a much more difficult problem. It is therefore advisable to examine the properties of the isoactinic by experiments specially arranged for the purpose, and to see whether the results obtained from such an investigation may be utilised in establishing the special isoactinic satisfying the conditions of the particular plate under observation.

§ 2. *General Theory of Photographic Photometry.*

Having thus defined the outline of the method, we proceed to develop the general theory leading to the principal equations of the problem. In the early days of actinic photometry the assumption was made that the actinic effect is proportional to the accumulated incident energy, I, acting on the sensitive film during the time of exposure, t, *i.e.*

$$A = It \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It is well known, now, that this relation cannot be reconciled with the observed facts. It involves the assumption that the *incident* energy, I, represents under all conditions the exact measure of the *effective* energy which is employed in the chemical processes inside the film. Modern researches, especially those made by Kron of the Potsdam Observatory,* have established the fact that the *effective* energy, which may be denoted by E, is a more complex function of the incident energy I.

Generally we may write

$$E \cdot \psi(I) = I \quad . \quad . \quad . \quad . \quad (3)$$

where $\psi(I)$ is a function of I only, but independent of t.

The actinic relation is then expressed by the equation

$$A = Et \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where the product Et represents the effective energy accumulated during the time of exposure t.

In a previous paper published in *M.N.*, **75**, 150, the character of the function $\psi(I)$ was examined in the light of the results of Kron's experiments. It was found that for a certain intensity I_0 which depends on the sensitiveness of the plate, $\psi(I)$ attains a minimum value, which, in accordance with equation (3), implies that this particular intensity is the most effective. Kron, for this reason, calls I_0 the "optimal" intensity of the plate.

* *Publikationen des astrophysikalischen Observatoriums zu Potsdam*, No. 67 : "Ueber das Schwärzungsgesetz photographischer Platten."

From equations (3) and (4) for the same value of A we obtain

$$\log I + \log t = \log \psi(I) + \text{constant.}$$

If t_0 is the time to be associated with the optimal intensity in order to produce the actinic effect A, and if we write

$$i = I/I_0\,;\ \tau = t/t_0,$$

then

$$\psi(I) = i\tau \qquad . \quad . \quad . \quad . \quad . \quad (5)$$

Kron's experimental results were shown to be satisfied by assigning to $\psi(I)$ the form

$$\psi(I) = \tfrac{1}{2}\left[i^{\alpha} + i^{-\alpha}\right] = i\tau \qquad . \quad . \quad . \quad (6)$$

It was further demonstrated that the exponent α does not materially depend on the sensitiveness of the plate, the results derived from fast and slow plates being satisfied by the same value of α, viz. 0·25.

The only essential difference between fast and slow plates lies in their values of I_0, the point of maximum effectiveness being reached in fast plates at a lower intensity than in slow plates.

Also, Kron's researches show that for plates of the same degree of sensitiveness, provided they are developed under exactly similar conditions, I_0 may be considered approximately constant.

This result is of great importance, since it shows that, if I_0 is determined for one particular plate, it is also known for all the plates of the *same batch*. It is this remarkable property which will enable us later on to determine the isoactinic for any particular plate, and, in conjunction with the time-scale, to establish the required intensity-scale.

The relation between the accumulated effective energies Et and the measured diameters or opacities depends on a number of conditions beyond our present knowledge, and therefore can only be studied empirically. Experience shows that the diameters of the same star exposed on different plates during the same interval of time are not exactly the same. This indicates that the same quantity Et produces different star images on different plates. In general the observed conditions appear to be satisfied by an equation of the form

$$\phi(D) = (Et)^{\sigma} \qquad . \quad . \quad . \quad . \quad (7)$$

where $\phi(D)$ is a function of the diameter (or opacity) and σ is a quantity which may be considered constant for any particular plate, but which is subject to variations from one plate to another.

Equations (3), (6), and (7) constitute the fundamental equations of actinometry. For practical purposes it is, however, more convenient to adopt logarithmic forms. We therefore write :

$$\log \phi(D) = \Phi(D) = \sigma \log E + \sigma \log t \qquad . \quad . \quad . \quad . \quad (8)$$

$$\log E = \log I - \log \psi(I) \qquad . \quad . \quad . \quad . \quad (9)$$

$$\log \psi(I) = \log i + \log \tau = \log \left[\frac{i^{0\cdot25} + i^{-0\cdot25}}{2}\right] \qquad . \quad (10)$$

Our next object will be to test these equations by the results obtained from photometric laboratory experiments and from astronomical observations.

§ 3. *Experimental Results regarding the Time-scale.*

If in equation (8) $E = E_c$ (constant), we obtain the general equation of the *time-scale*

$$\frac{1}{\sigma}\Phi(D) = \log t + \log E_c = \log t + \text{constant}.$$

This equation shows that time-scales derived for different energies E_c on the same plate are similar in form. When represented by curves with co-ordinates D and $\log t$, the curves obtained for different values of E_c are therefore alike, and differ only with regard to the zero-point of the $\log t$ co-ordinate.

In the paper in *M.N.*, **75**, 150, this important property of the time-scales was shown to be in accordance with Kron's experiments and also with the evidence of the astronomical observations made at this Observatory. It had already been pointed out by Sir William Abney in his important investigation published in the *Photographic Journal*, **18**, 320 (see his table iv. on p. 304).

The gradation of the time-scale depends on the quantity σ. For two different plates we get

$$\left.\begin{aligned}\phi(D) &= \sigma_m \log t + a_m\\ \phi(D) &= \sigma_n \log t + a_n\end{aligned}\right\} \quad . \quad . \quad . \quad (11)$$

Hence, if the time-scale for one plate is completely known, that of any other plate may be found, provided that at least two exposures have been made. Suppose that the exposures on the second plate are t_1 and t_2, and the corresponding diameters of the same star D_1 and D_2. Then, with these diameters we obtain from the known time-scale of the first plate the corresponding values $[\log t]_1$ and $[\log t]_2$, and, according to (11),

$$\sigma_n \log t_1 + a_n = \sigma_m[\log t]_1 + a_m$$
$$\sigma_n \log t_2 + a_n = \sigma_m[\log t]_2 + a_m.$$

Disregarding the a terms, which for the present may be left arbitrary, we have

$$\frac{\sigma_m}{\sigma_n} = \frac{\log t_1 - \log t_2}{[\log t]_1 - [\log t]_2} \quad . \quad . \quad . \quad (12)$$

and the time-scale of the second plate, apart from the arbitrary constant, is found from the time-scale of the first plate by adopting

$$\log t = \frac{\sigma_m}{\sigma_n}[\log t] + \text{constant} \quad . \quad . \quad . \quad (13)$$

This property of the time-scale has been tested by comparing the results derived from different plates on which exposures of the South Polar Sequence were made.

Table I. shows in the first two columns the mean co-ordinated values of D and $\log t$ derived from the exposures of a star of magnitude 6·17 on nine plates. The observed time-scales of two individual plates derived from the same star are shown in columns 3, 4 and 6, 7. Columns 5 and 8 exhibit the values of $\log t$ which are obtained when the $[\log t]$ of the mean time-scale referring to corresponding diameters are multiplied by the factors 0·90 and 1·13 respectively. The agreement between columns 4, 5 and 7, 8 shows the property of the time-scales here discussed.

TABLE I.

		Plate X.			Plate XVII.		
D.	$[\log t]$.	D.	$\log t$.		D.	$\log t$.	
			Obsd.	Compd.		Obsd.	Compd.
1·30	3·81						
1·20	3·47	1·20	3·12	3·12			
1·10	3·13	1·10	2·82	2·82			
1·00	2·79	1·00	2·52	2·51	1·00	3·14	3·15
0·90	2·44	0·90	2·22	2·20	0·90	2·75	2·76
0·80	2·09	0·80	1·90	1·88	0·80	2·36	2·36
0·70	1·72	0·70	1·56	1·55	0·70	1·93	1·94
0·60	1·32	0·60	1·20	1·19	0·60	1·48	1·49
0·50	0·88	0·50	0·79	0·79	0·50	1·00	1·00
0·40	0·39	0·40	0·35	0·35	0·40	0·44	0·44

Similar results were obtained from an examination of the time-scales published in Abney's paper (*Photographic Journal*, **18**). It was found as before that the values of $\log t$ in the individual time-scales are related to the $[\log t]$ of a mean time-scale corresponding with the same actinic effects (opacities in this case) by the equation

$$\log t = a[\log t] + b \quad . \quad . \quad . \quad . \quad (14)$$

where a and b represent constants peculiar to the plate. From nine different plates of varying sensitiveness the average error of a single value of $\log t$ computed from equation (14), as compared with that given by Abney, was found $\pm 0{\cdot}02$, which roughly corresponds with an error $\pm 0^{m}{\cdot}04$.

The time-scales for different brands of plates observed by Kron, and published in table xviii. of his paper, show the same property. If, for instance, the scales for the two fast plates *Schleussner* and *Seed* 27 are compared with that of the slowest plate *Afga-*

Diapositiv for the same actinic effects (Schwärzung), the following figures are obtained :—

Actinic Effect.	Fast Plate. $\log t_f$.	Slow Plate. $\log t_s$.	Residuals. $(o-c)$.
64	+1·035	+0·348	−0·007
62	+0·865	+0·276	−0·002
60	+0·699	+0·205	+0·001
58	+0·531	+0·133	+0·005
56	+0·360	+0·059	+0·007
54	+0·189	−0·018	+0·007
52	+0·022	−0·100	0·000
50	−0·145	−0·182	−0·007
48	−0·320	−0·262	−0·008
46	−0·522	−0·351	−0·007
44	−0·760	−0·442	+0·009

A representation of the values $\log t_s$ by the equation

$$\log t_s = 0{\cdot}449 \log t_f - 0{\cdot}110$$

leaves the insignificant residuals shown in the fourth column.

Hence the result that if a time-scale is once obtained, the time-scales of other plates taken under similar conditions of observation may be derived, if on these plates the star images are recorded for one long and one short exposure only.

§ 4. *Experimental Results regarding the Isoactinics.*

If different intensities, I, are exposed during such times, t, that the diameters (or opacities) are the same, equation (8) takes the form

$$\left.\begin{aligned} \log \mathrm{E} + \log t &= \text{constant} \\ \text{or} \\ \log \psi(\mathrm{I}) &= \log i + \log \tau \\ \text{where} \\ i &= \mathrm{I}/\mathrm{I}_0\,;\ \tau = t/t_0 \end{aligned}\right\} \quad . \quad . \quad . \quad (15)$$

I_0 being the optimal intensity of the plate, and t_0 the time during which it must be exposed in order to produce the required actinic effect.

The character of the function $\psi(\mathrm{I})$, derived from Kron's experiments, has already been shown in equation (10); for a fuller discussion the paper in *M.N.*, **75**, may be consulted. Considering the photometric relation

$$\log i = -0{\cdot}4(m - m_0),$$

where m_0 represents the magnitude corresponding with the optimal intensity I_0, equation (10) may be written in the form

$$\log \tau = 0{\cdot}4(m - m_0) + \log \tfrac{1}{2}\left[10^{\ 1(m-m_0)} + 10^{0{\cdot}1(m_0-m)}\right] \quad . \quad (16)$$

The correlated values of $\log \tau$ and $(m - m_0)$ satisfying equation (16) are shown in Table II.

TABLE II.

$\log\tau$.	$(m-m_0)$.	$\log\tau$.	$(m-m_0)$.	$\log\tau$.	$(m-m_0)$.	$\log\tau$.	$(m-m_0)$.
−1·50	−4·19	+0·50	+1·21	+2·50	+5·54	+4·50	+ 9·59
−1·40	−3·89	+0·60	+1·44	+2·60	+5·74	+4·60	+ 9·79
−1·30	−3·59	+0·70	+1·67	+2·70	+5·95	+4·70	+ 9·99
−1·20	−3·29	+0·80	+1·90	+2·80	+6·16	+4·80	+10·20
−1·10	−2·99	+0·90	+2·12	+2·90	+6·36	+4·90	+10·40
−1·00	−2·70	+1·00	+2·35	+3·00	+6·56	+5·00	+10·60
−0·90	−2·41	+1·10	+2·57	+3·10	+6·77	+5·10	+10·80
−0·80	−2·13	+1·20	+2·79	+3·20	+6·97	+5·20	+11·00
−0·70	−1·85	+1·30	+3·01	+3·30	+7·17	+5·30	+11·20
−0·60	−1·57	+1·40	+3·23	+3·40	+7·37	+5·40	+11·40
−0·50	−1·30	+1·50	+3·44	+3·50	+7·58	+5·50	+11·60
−0·40	−1·03	+1·60	+3·66	+3·60	+7·78	+5·60	+11·80
−0·30	−0·77	+1·70	+3·87	+3·70	+7·98	+5·70	+12·00
−0·20	−0·51	+1·80	+4·08	+3·80	+8·18	+5·80	+12·20
−0·10	−0·25	+1·90	+4·29	+3·90	+8·38	+5·90	+12·40
0·00	0·00	+2·00	+4·50	+4·00	+8·59	+6·00	+12·60
+0·10	+0·25	+2·10	+4·71	+4·10	+8·79	+6·10	+12·80
+0·20	+0·49	+2·20	+4·92	+4·20	+8·99	+6·20	+13·00
+0·30	+0·73	+2·30	+5·12	+4·30	+9·19	+6·30	+13·20
+0·40	+0·97	+2·40	+5·33	+4·40	+9·39	+6·40	+13·40

An indication of the accuracy with which Kron's observations are represented by equation (16) may be gathered from Table IIa., where the observed correlated values of $\log\tau$ and $(m-m_0)$, first for the combination of fast and slow plates, and secondly for fast plates only, are shown separately in the first two columns of each set, while the third columns give the values of $(m-m_0)$, corresponding with the $\log\tau$ of the first columns, and taken from Table II.

TABLE IIa.

Fast and Slow Plates combined:				Fast Plates only:			
	$(m-m_0)$.				$(m-m_0)$.		
$\log\tau$.	Obsd.	Table II.	Diff.	$\log\tau$.	Obsd.	Table II.	Diff.
−1·76	−4·82	−5·00	+0·18	−1·65	−4·54	−4·65	+0·11
−1·26	−3·41	−3·46	+0·05	−1·23	−3·36	−3·38	+0·02
−0·76	−2·02	−2·01	−0·01	−0·77	−2·07	−2·04	−0·03
−0·24	−0·61	−0·61	0·00	−0·23	−0·59	−0·59	0·00
+0·24	+0·60	+0·59	+0·01	+0·29	+0·73	+0·71	+0·02
+0·77	+1·83	+1·83	0·00	+0·78	+1·88	+1·86	+0·02
+1·30	+3·04	+3·01	+0·03	+1·28	+2·97	+2·97	0·00
+1·77	+4·08	+4·02	+0·06	+1·72	+3·92	+3·91	+0·01
+2·20	+4·91	+4·92	−0·01	+2·19	+4·90	+4·91	−0·01
+2·89	+6·37	+6·35	+0·02	+2·90	+6·40	+6·36	+0·04

After the publication of the paper in vol. **75** of the *M.N.*, the author's attention was drawn to the investigations made in 1894 by Sir William Abney (*Photographic Journal*, **18**), which have proved extremely valuable as a test of the results obtained by Kron. Abney operated with light sources of highly different intensities (amyl-acetate lamp, electric light, and electric spark). On each plate he established first a time-scale, and secondly an intensity-scale, over a limited range of relative intensities. Hence we are enabled to examine the character of the function $\psi(I)$ in comparatively short branches, but at different distances from its minimum point.

Under these conditions $\psi(I)$ may be represented with sufficient accuracy by an equation of the form

$$\left.\begin{aligned} \log \psi(I) &= -q \log I + \text{constant} \\ \text{where} \qquad q &= 0{\cdot}25 \frac{i_m^{-0{\cdot}25} - i_m^{+0{\cdot}25}}{i_m^{-0{\cdot}25} + i_m^{+0{\cdot}25}} \end{aligned}\right\} \quad . \quad . \quad (17)$$

and $i_m = I_m/I_0$, I_m being the intensity corresponding with the centre of the branch of the $\psi(I)$ curve.

Substituting in (8), we have

$$\phi(D) = \sigma(1+q) \log I + \sigma \log t \; . \quad . \quad . \quad (18)$$

The equation of the time-scale ($I = I_c$) is therefore

$$(D) = \sigma(1+q) \log I_c + \sigma \log t \; . \quad . \quad . \quad (19)$$

and that of the intensity-scale ($t = t_c$)

$$\phi(D) = \sigma(1+q) \log I + \sigma \log t_c \; . \quad . \quad . \quad (20)$$

For equal actinic effects, I and t are connected by the equation

$$\log t = (1+q) \log I + \text{constant} \; . \quad . \quad . \quad (21)$$

from which q can be calculated.

In another set of Abney's experiments, in place of the intensity-curve, the intensities, I, and times of exposure, t, were so chosen that the product It was constant, the time-scale being observed as before. By comparing equal actinic effects, we obtain in this case the relation

$$\log t = q \log I + \text{constant} \quad . \quad . \quad . \quad (22)$$

With regard to q, Kron's experiments, according to equation (17), show that for very large value of i_m, $q = -0{\cdot}25$, and for very small values, $q = +0{\cdot}25$, while for $i_m = 1$, $q = 0$.

A general discussion of Abney's experiments, on the other hand, led to the following results:—

	Amyl-acetate Lamp.	Electric Light.
For slow plates . .	$q = +0{\cdot}24$	$q = -0{\cdot}09$
For fast plates . .	$q = -0{\cdot}05$	$q = -0{\cdot}26$

The range in the values of q from $+0{\cdot}24$ to $-0{\cdot}26$ agrees well with that indicated in Kron's experiments. Further, for slow plates the optimal intensity $I_0(q = 0)$ lies between the intensity of the amyl-acetate lamp and that of the electric light, which is about 12,000 times that of the lamp. On the other hand, for fast plates I_0 is less than the intensity of the lamp. Abney's experiments thereby confirm those of Kron, showing that the optimal intensity of slow plates is greater than that of fast plates.

While the optimal intensity, I_0, or its corresponding magnitude, m_0, is considerably influenced by the sensitiveness of the plate and also by the conditions of developing, Kron's experiments justify, however, the assumption that I_0 varies very little for plates of the same brand and developed under similar conditions. As already mentioned, this result is of vital importance in the method of determining photographic magnitudes which is proposed later on.

§ 5. *The Relation between Time and Intensity derived from Astronomical Observations.*

For certain regions of the sky, such as the two poles, for instance, photographic magnitudes extending over a considerable range are now available. These may be utilised for establishing a relation between t and m similar to that shown in Table II., which was based on the laboratory experiments of Kron.

Let the stars in such a standard region be exposed successively during varying intervals of time, t_1, t_2 . . . , generally t_c, say. Each star gives a separate determination of the time-scale, which may be expressed in the form of a graph whose co-ordinates are the measured diameters D and the logarithms of the time of exposure. In accordance with the property of the time-scale discussed above, the individual curves may be combined into one curve, referring to any selected magnitude, m_c, by appropriate shifts along the $\log t$ co-ordinate. In this manner partially overlapping curves may be welded into one time-scale applicable to the whole range of the plate.

For each measured diameter whose time of exposure is t_c, the time-scale is next entered, and the corresponding value of $\log t$ is recorded. Obviously the values $\log t_1/t$, $\log t_2/t$ . . . ., $\log t_c/t$ for one and the same star must agree within the limits of the errors of observation. Forming the mean values of $\log t_c/t$, every star of known magnitude yields co-ordinated values of $\log t_c/t$ and m.

Now, according to the general equation (8), we have

$$\phi(D) = \sigma \log E_c + \sigma \log t \text{ (time-scale)}$$
$$\phi(D) = \sigma \log E + \sigma \log t_c \text{ (intensity-scale).}$$

Comparing equal diameters, we find therefore

$$\log t_c/t = \log E_c/E = \log E_c - \log I + \log \psi(I)$$
$$= \log E_c - \log I_0 + \log \tau,$$

according to equations (9) and (10). Since E_c and I_0 are constant, it follows that

$$\log t_c/t = \log \tau + \text{constant}.$$

Hence the co-ordinated values $\log t_c/t$ and m differ from $\log \tau$ and $(m - m_0)$ of Table II. by *constants* only.

As an example, photographs of the brighter stars of the Pleiades are selected. Photographic magnitudes of these stars, obtained by means of a diffraction grating, were published by Professor Hertzsprung in *Ast. Nach.*, 4452. In addition, their visual magnitudes have been observed at Potsdam, which by means of the effective wave-lengths determined by Hertzsprung may be reduced to the photographic scale. In Table III. the mean values of $\log t_c/t$ and of the photographic magnitudes (Hertzsprung and Potsdam) are shown for nine groups of stars. Assuming $m_0 = 3{\cdot}09$ and entering Table II. with the argument $(m - m_0)$, the corresponding values of $\log \tau$ are shown in the fifth column. The differences $(\log t_c/t - \log \tau)$ are indeed as nearly constant as the accuracy of the observations would lead us to expect.

Table III.

Group.	m_p (Obsd.).	$\log t_c/t$ (Obsd.).	No. of Stars.	$\log \tau$ (Table II.).	$(\log t_c/t - \log \tau)$.
I.	3·71	+0·01	5	0·25	−0·24
II.	5·19	+0·66	4	0·89	−0·23
III.	6·55	+1·23	5	1·51	−0·28
IV.	6·98	+1·50	5	1·71	−0·21
V.	7·47	+1·70	5	1·94	−0·24
VI.	7·88	+1·87	5	2·14	−0·27
VII.	8·33	+2·07	5	2·35	−0·28
VIII.	8·80	+2·34	5	2·58	−0·24
IX.	9·09	+2·48	4	2·72	−0·24

If m_0 is once determined for a plate of a particular brand, its value may be accepted for other plates of the same brand under similar conditions of development. Hence only one unknown constant remains to be determined, which may be readily found if the magnitude of *one* star on the plate is known from other sources.

As an example, the photographic magnitudes of groups of stars of the South Polar Sequence may be determined in this manner. In Table IV. the values of $\log t_c/t$ found for two different sets of plates, marked A and B, containing six and four plates respectively, are shown in columns 2 and 3. The time-scales from which these values were derived refer to different stars, hence the constant difference between the two sets. Now let it be supposed that we know the photographic magnitude of Group I., viz. 6 60, and also the value $m_0 = 3{\cdot}09$.

Entering with $m - m_0 = 3{\cdot}51$ into Table II., we find the corresponding value of $\log \tau = 1{\cdot}53$.

Hence for A

$$\log \tau = \log t_c/t + 1{\cdot}32,$$

for B

$$\log \tau = \log t_c/t + 1{\cdot}85.$$

Entering with $\log \tau$ into Table II., we find the corresponding values of $(m - m_0)$, and finally, by adding 3·09, the photographic magnitudes of columns 5 and 6.

TABLE IV.

Group.	$\log t_c/t$.		No. of Stars.	m.		Mean m.	
	A.	B.		A.	B.	($m_0 = 3{\cdot}09$).	($m_0 = 2{\cdot}00$).
I.	+0·21	−0·32	3	6·60	6·60	6·60	6·60
II.	+1·20	+0·73	5	8·67	8·79	8·73	8·72
III.	+1·76	+1·27	7	9·82	9·88	9·85	9·83
IV.	+2·10	+1·58	8	10·50	10·52	10·51	10·49
V.	+2·24	+1·74	11	10·79	10·85	10·82	10·79
VI.	+2·43	+1·91	13	11·16	11·19	11·18	11·15
VII.	+2·62	+2·08	18	11·55	11·53	11·54	11·51
VIII.	+2·76	+2·25	12	11·84	11·88	11·86	11·82

The supposition made in these calculations, viz. that m_0 is the same as in the case of the Pleiades, can, of course, not be expected to be strictly fulfilled. It is important, therefore, to examine the influence of an error in the assumed value of m_0. For this purpose the magnitudes of the eight groups of Table IV. were also computed on the assumption $m_0 = 2{\cdot}00$. The resulting figures are shown in the eighth column. Comparing columns 7 and 8, the differences are found to be almost negligible. Hence small fluctuations of m_0 from plate to plate appear to have no serious influence on the derived magnitudes. Since Kron's results show that the changes of m_0 from plate to plate for the same brand submitted to the same treatment in developing are well within the limit here considered, the assumption of a constant m_0 for all plates is therefore justified.

This, however, supposes that the photographs are taken with the same instrument. Since the energy acting on a certain point of the film depends on the amount of starlight concentrated by the collecting lens on this point, m_0 is a function of the aperture of the telescope. It is therefore necessary to determine m_0 for each instrument separately by a series of experimental plates exposed on a standard region and treated in the manner described above. Where a diffraction grating or a standardised screen mounted before the object-glass are available, m_0 can be determined by means of a single star of known magnitude by a method fully described in the paper in *M.N.*, **75**.

The determination of photographic magnitudes by the method here proposed depends essentially on the relation between τ and m shown in Table II. So far as investigations made at this Observatory go, the accuracy of this table seems to be established

for values of $(m - m_0) < 9^{m}\cdot 0$. It is highly desirable, however, that its competence should be still further examined for plates of different emulsions by means of the stars of the North Polar Sequence, for which reliable photographic magnitudes down to the 17th are available.

§ 6. *Proposed Method of determining Photographic Magnitudes.*

It is supposed—

(*a*) that all the observations are made with the same instrument;

(*b*) that plates of the same batch are used throughout;

(*c*) that the plates are developed under exactly the same conditions as regards the nature, degree of concentration, and temperature of the developer.

I. *Exposures on Standard Region.*—A region containing a sufficient number of stars of known magnitudes covering as wide a range as possible is selected (the North or South Polar region, for instance). On one and the same plate successive exposures of varying duration (from 1000^{s} to 10^{s}, say) are given, and the diameters of all measurable star images are micrometrically determined. The time-scales given by individual stars are combined into a single time-scale by the method above described.

From this mean time-scale the values of $\log t$ corresponding with all the measured diameters are determined, and the quantities $\log t_c/t$ are computed, t_c being the time of exposure referring to the diameter in question.

All the values of $\log t_c/t$ referring to the same star are then combined into means. The resulting mean values of $\log t_c/t$ as functions of the known magnitudes, m, can be represented in the form of a table, which may be denoted as Table A, and which takes the place of Table II. We thus obtain from the standard region (*a*) a mean time-scale, and (*b*) the relation between m and $\log t_c/t$, and these two relations once established may be used for the derivations of the magnitudes on any other plate of the same emulsion.

II. *Exposures on any Region whose Magnitudes are to be determined.*—It is supposed that at least one long and one short exposure are recorded, their durations being t_1 and t_2.

With the corresponding diameters D_1 and D_2 of any star the mean time-scale (standard region) is entered, the correlated values of $\log t$ being denoted by $[\log t]_1$ and $[\log t]_2$. Then equation (12) gives $\frac{\sigma_m}{\sigma_n}$, and the true time-scale of the plate apart from a constant is obtained by multiplying the $\log t$ of the *mean* time-scale into this factor.

Having derived the time-scale of the plate, the values of $\log t$ corresponding with all the measured diameters are found, and for every star the quantity $\log t_c/t$ is determined. For stars of the

same magnitude these quantities differ from those shown in Table A by a constant a, say.

If, therefore, the magnitude of one star on the plate is known, and the value $\log t_c/t$ corresponding with this magnitude is taken from Table A, the difference between this and the observed value is equivalent to the constant a, *i.e.*

$$\log t_c/t \text{ (Plate)} + a = \log t_c/t \text{ (standard region)}.$$

Entering, for every star, Table A with the quantities on the right-hand side of this equation, the final values of m are directly obtained.

There is only one unknown, viz. the zero-point from which the magnitudes are reckoned, which cannot be ascertained by the internal evidence of the plates themselves; and hence the knowledge of the magnitude of at least one star from extraneous sources is necessary. The visual magnitude of a bright star and its type-correction are sufficient for the purpose.

The method is applicable to the derivation of the photographic magnitudes of the Astrographic Catalogues by utilising the long and short exposures recorded on their plates for establishing the time-scales of the individual plates. The only additional observations required are some exposures on a standard region on plates of the same sensitiveness, from which the mean time-scale and the relation between m and $\log t_c/t$ (isoactinic) are to be derived.

§ 7. *Alternative Method applicable to Faint Stars.*

The method of deriving magnitudes by means of the time-scale is open to criticism on account of the fact that the exposures have to be spread over a considerable time during which atmospheric conditions are supposed to remain unaltered.

Systematic errors in the observed time-scales arising from this source would necessarily affect the relation between the quantities m and $\log t_c/t$. The result would be that, for instance in the two sets A and B of Table IV., the differences $\Delta \log t_c/t$ would not be strictly constant throughout the eight groups. The fact, however, that in this respect the results derived from individual plates are remarkably consistent, points to the conclusion that the time-scales were correctly determined.

Nevertheless it is of interest to notice that, at least for faint stars ($m > 7$ about), which are of special importance for statistical investigations, magnitudes may be determined *without* the aid of a time-scale, provided that standardised magnitudes of one particular region (North or South Polar Sequence) are known.

For stars whose intensities are considerably less than the optimal intensity I_0, the simplified general equation (18) may be adopted, viz.

$$\phi(D) = \sigma(1+q)\log I + \sigma \log t.$$

Suppose the standard region to be exposed on a plate during the time t_c. The relation between D and I is then of the form

$$\phi(D) = \sigma(1+q)\log I + \sigma \log t_c = -0{\cdot}4\sigma(1+q)m + a \quad . \quad (23)$$

where a is a constant for all stars.

The magnitudes m being known, the relation (D, m) may be represented in the form of a graph with co-ordinates D, m. This graph will be called the *standard intensity-scale.* Let now the plate be exposed again during the same interval of time t_c, but with a screen in front of the object-glass by which the magnitudes are increased by an unknown amount Δm.

Obviously, denoting by D_1 and D_2 the diameters of the images of the same star without and with screen, then taking with these diameters the corresponding magnitudes m_1 and m_2 from the graph (D, m), we have

$$m_2 - m_1 = \Delta m.$$

Let us now expose another plate, not necessarily but preferably of the same brand, on any region for which the magnitudes are required, during the same interval of time t_c, two exposures of the same duration being given with free and screened object-glass. The intensity-scale for this plate is then represented by an equation of the form

$$\phi(D) = -0{\cdot}4\sigma'(1+q')m + a' \quad . \quad . \quad . \quad (24)$$

If now, with the measured diameters D_1 and D_2 of the same star, we take from the standard intensity-scale the corresponding values of m, which will be denoted by $[m]_1$ and $[m]_2$, we have

$$\frac{\Delta m}{[m]_2 - [m]_1} = \frac{\sigma'(1+q')}{\sigma(1+q)} = b, \text{ say} \quad . \quad . \quad . \quad (25)$$

and

$$m = b[m] + \text{constant}.$$

Hence, apart from the zero-point of the m co-ordinate, the intensity-scale (D, m) of the plate is obtained from that of the standard region simply by multiplying the magnitudes of the latter into the factor b derived from equation (25). The photographic magnitude of one star known from other sources is sufficient to fix the zero-point.

It will be noticed that this method dispenses with the time-scale altogether, and rests on the sole assumption that Kron's isoactinic may be considered a straight line within the limits of intensities recorded on the plate. This condition is very approximately fulfilled on ordinary plates for the feeble intensities with which we are dealing in star-investigations.

§ 8. *Summary and Comparison between the Present and Older Theories.*

The general relation between the observed diameters (or opacities) D, the incident intensities I, and the times of exposure

t, on which the preceding method is based, is contained in the equation

$$\phi(\mathrm{D}) = \left[\frac{\mathrm{I}t}{\psi(\mathrm{I})}\right]^{\sigma} \quad . \quad . \quad . \quad . \quad (26)$$

where σ is a constant for any given plate, but is subject to changes from one plate to another, and where the function $\psi(\mathrm{I})$ can be empirically represented in the form

$$\psi(\mathrm{I}) = \tfrac{1}{2}\left[i^{0\cdot25} + i^{-0\cdot25}\right]; \; i = \mathrm{I}/\mathrm{I}_0 \quad . \quad . \quad . \quad (27)$$

I_0 being the *optimal* intensity of the plate, at which $\psi(\mathrm{I})$ reaches the minimum value unity. The validity of equation (26) has been tested by the laboratory experiments made by Abney and Kron and by an extensive series of astronomical observations made at this Observatory.

The point in regard to which equation (23) differs from earlier analytical expressions which were adopted for the general relation between D, I, and t is the form of the function $\psi(\mathrm{I})$.

In the early stages of photographic photometry the assumption was made that

$$\psi(\mathrm{I}) = \text{constant},$$

and hence

$$\phi(\mathrm{D}) \propto [\mathrm{I}t]^{\sigma},$$

which leads to the simple form of the isoactinic

$$\mathrm{I}t = \text{constant} \quad . \quad . \quad . \quad . \quad (28)$$

known as the *reciprocity law* (first enunciated by Bunsen).

The inadmissibility of this assumption was soon recognised. The next important step in the development of the photometric theory of the plate was taken by Schwarzschild, who introduced the function $(\psi)\mathrm{I}$ in a form which is applicable within comparatively narrow limits of I. If only a short branch of $\psi(\mathrm{I})$, centred on an intensity I_m, say, is considered, we may write approximately

$$\psi(\mathrm{I}) \propto \mathrm{I}^{-q},$$

where

$$q = 0\cdot25\,\frac{i_m^{-0\cdot25} - i_m^{+0\cdot25}}{i_m^{-0\cdot25} + i_m^{+0\cdot25}}, \text{ and } i_m = \mathrm{I}_m/\mathrm{I}_0 \text{ (see equation (17)).}$$

Hence equation (26) takes in this case the form

$$\phi(\mathrm{D}) \propto \left[\mathrm{I}^{1+q} . t\right]^{\sigma} \quad . \quad . \quad . \quad . \quad (29)$$

and the isoactinic is expressed by

$$\mathrm{I}^{1+q} . t = \text{constant}$$

or

$$\mathrm{I} . t^{p} = \text{constant}, \; p = \frac{1}{1+q} \quad . \quad . \quad (30)$$

which is the form adopted by Schwarzschild.

Kron's experiments have for the first time demonstrated the general form of the function $\psi(I)$. They have further shown that this form may for practical purposes be assumed to be the same for plates of all degrees of sensitiveness. The only quantity involved which appears to depend on the internal properties of the plate is the optimal intensity I_0. But Kron's experiments make it abundantly clear that for plates of the same brand developed under similar conditions I_0 is practically constant. This important property enabled us to frame the preceding method, by which the photographic magnitudes, on a true photometric basis, can be derived for all the stars of *any* given plate, provided the magnitude of one star is known from other sources.

The following seem to be the most suitable forms of the general photometric equations of the photographic plate:

$$\left.\begin{aligned} \phi(D) &= (et)^{\sigma} \\ \frac{2}{e} &= \left[10^{0\cdot3(m-m_0)} + 10^{0\cdot5(m-m_0)}\right] \end{aligned}\right\} \quad . \quad . \quad (31)$$

where σ and m_0 represent constants peculiar to the plate. The letter D stands for the observed actinic effect expressed in terms either of the sizes or opacities of the images.

On Professor Sampson's "Note on the Southern Magnitude Distribution." By J. Halm.

In *Monthly Notices*, 77, 613, Professor Sampson discusses the magnitudes of the Perth Astrographic zone $-32°$ with the view of testing the relation between star numbers and magnitudes established by Messrs. Chapman and Melotte for northern stars. He arrives at the conclusion "that the Chapman and Melotte numbers do not in the least represent the actual distribution in the Perth zone $-32°$."

Since this verdict is in direct opposition to the result which the author obtained from a discussion of the C.P.D. magnitudes in the same region, and more especially since such a divergence of conclusions affects most vital issues in the more general statistical problems dealing with the structure of the universe, it is important to trace the cause of this discrepancy.

The cardinal point of Professor Sampson's argument seems to be contained in the statement that, if Chapman and Melotte's results were accepted, "the limit of the Cordoba observers, which they give as $9\frac{1}{2}$ mag., with a few of 10 mag., should really be $10\frac{1}{2}$ mag. or more. But as they used only a 5-inch transit-circle, I find it very hard to suppose that they habitually observed $10\frac{1}{2}$ mag. stars, with occasional 11 mag."

Apparently Professor Sampson is under the impression that the estimates of brightness given by the Cordoba observers represent magnitudes in the photometric sense. This, however, is far from being the case. Let us take the case of Thomé's Cordoba Durchmusterung, whose observations were made with an instrument of little less than 5 inches aperture, the magnitudes being estimated on the same plan as adopted in Gould's zones. In the *Annals of the Harvard College Observatory*, **72**, No. 7, the system of the C.D.M. (Cordoba Durchmusterung) is compared with the Harvard photometric system (visual). The mean results quoted there for stars contained in vols. **16** and **17** of the C.D.M. refer approximately to the zone $-32°$, and are therefore comparable with the material of the Perth zone.

Now from Table XI., on p. 242, *l.c.*, the following corrections (Harvard photometric–C.D.M.) are obtained:—

C.D.M. Mag.	(Harvd.–C.D.M.)	C.D.M. Mag.	(Harvd.–C.D.M.)
8·0	+0·1	9·4	+1·0
8·5	+0·4	9·5	+1·2
8·6	+0·5	9·6	+1·3
9·0	+0·6	9·7	+1·4
9·1	+0·7	9·8	+1·6
9·2	+0·8	9·9	+1·7
9·3	+0·9	10·0	+1·9

A star estimated 9·5 by the Cordoba observers is, therefore, actually of the 10·7th magnitude on the photometric visual scale, and a star estimated 10·0 is of magnitude 11·9. Table XI. also shows that about 18 per cent. of all the stars observed are fainter than mag. 11·0 (Harvard visual), and about 9 per cent. fainter than $11^{m}·6$.

Hence the Cordoba observers were actually in the habit of observing with their 5-inch instruments stars down to mag. 10·7, and not, as Professor Sampson assumes, to only 9·5.

We may, however, get more reliable quantitative results showing the correctness of Chapman and Melotte's figures in the following manner. Professor Sampson quotes the values of Mr. Cooke's comparison images A, B, C . . . expressed on the C.P.D. scale as follows:—

A = 8·4 C.P.D.	E = 9·7 C.P.D.
B = 8·7	F = 10·0
C = 8·8	G = 10·2
D = 9·3	H = 10·3

Now, according to Professor Kapteyn (Introduction to vol. **1** of C.P.D., p. 49), the difference (C.D.M.–C.P.D.) for the belt $-31°·5$

is $-0^m\cdot3$, and is practically the same for all magnitudes within the range of the C.P.D. We thus find:

TABLE I.

	C.P.D.	C.D.M.	Harvd. (visual).
A	8·4	8·1	8·2
B	8·7	8·4	8·8
C	8·8	8·5	9·0
D	9·3	9·0	9·7
E	9·7	9·4	10·4
F	10·0	9·7	11·1
G	10·2	9·9	11·6
H	10·3	10·0	11·9

We next consider Professor Turner's magnitude-equivalents derived from Chapman and Melotte (*M.N.*, **75**, 144). First we have to bear in mind that these magnitudes are *photographic*, and, for this reason, differ from the visual magnitudes by the average colour-index of the stars of the brightness here considered. This colour-index, so far as type is concerned, may be found from Miss Fleming's lists of selected areas published in *Harvard Annals*, **71**, No. 2. For the zone $-45°$ it amounts to $+0\cdot55$, and is approximately constant between magnitude 6·5 and 9·3. Hence, in order to convert Professor Turner's *photographic* magnitudes (according to Chapman and Melotte) into *visual* magnitudes, 0·55 has to be deducted.

Secondly, account must be taken of the fact that the magnitudes of Chapman and Melotte are based on the provisional system of the North Polar Sequence, published in *Harvard Circular*, No. 170. This system, according to the Mount Wilson observations, requires the following corrections:—

Mag.	(Mt. Wilson–H.C. 170).	Mag.	(Mt. Wilson–H.C. 170).
8·0	+0·09	11·0	+0·29
9·0	+0·18	12·0	+0·31
10·0	+0·25	13·0	+0·32

Hence, from Professor Turner's figures we obtain:

TABLE II.

	Photogr. Mag.		Vis. Mag. (Harvard).
	C. & M.	Mt. Wilson.	
A	8·55	8·69	8·1
B	9·05	9·23	8·7
C	9·64	9·87	9·3
D	10·08	10·33	9·8
E	10·64	10·91	10·4
F	11·20	11·49	10·9
G	11·76	12·07	11·5
H	12·25	12·56	12·0

The last column is obtained by subtracting 0·55 from the Mount Wilson photographic magnitudes.

The remarkable agreement between the last columns of Tables I. and II. must be considered as a conclusive proof of the fact that the distribution of stars in the Perth zone -32° is satisfactorily represented by the Chapman and Melotte numbers.

The actinic effects A, B, C . . . used by Mr. Cooke as comparison scale represent equal steps of a time-scale photographed on a special plate. Professor Sampson converts this time-scale into a magnitude-scale by using Schwarzschild's isoactinic relation. This procedure, however, is only warranted in comparisons made on the *same* plate, but cannot be allowed in a case like the present, where a time-scale of a special plate is compared with the mean magnitude-scale of the whole Catalogue, for the simple reason that the gradations of both time- and magnitude-scale are subject to changes from plate to plate. On this account the magnitude step $0^m{\cdot}3$ derived by Professor Sampson, instead of about 0·54 shown in the above tables, is open to criticism.

[An instructive example illustrating the different gradations of the time-scales on different plates is shown by the comparison plates used in the investigations of the photographic magnitudes of the North Polar Sequence which are published in the *Harvard Annals*, **71**, No. 3. The star images on these plates were obtained from specially selected stars by exposures of 3^s, 9^s, 27^s, etc. With Schwarzschild's actinic law ($q = 1{\cdot}25$) the magnitude step between successive exposures should be $2 \times 0{\cdot}477 = 0^m{\cdot}95$. The steps actually derived from comparisons with standard magnitudes are shown in table xviii. on p. 120, *ibid.* The mean of the 13 plates gives 0·93, which is in good agreement with the computed value. But the individual plates exhibit very considerable fluctuations ranging from 0·66 to 1·38. This clearly shows how little confidence can be placed in a magnitude-scale derived from the time-scale of a single comparison plate by means of Schwarzschild's law, if applied to stars on *other* plates. A discussion of the properties of the time-scales is given in the author's paper "On the Determination of Photographic Magnitudes" (second paper).]

Having shown that the statistical evidence furnished by the Perth zone in its *entirety* confirms Chapman and Melotte's figures, we may proceed to examine the material more in detail for special regions of the zone.

In a paper recently sent to the Society, entitled "On the Relation between Star Counts and Apparent Magnitudes," the author has shown that, according to the statistical results obtained by Chapman and Melotte, the relation (B_m, m) in any region of the sky can be represented by the equation

$$m - \bar{M}_0 = 2{\cdot}98 + \tfrac{5}{3} \log B_m + A \,.\, B_m^{0{\cdot}183},$$

where

$$A = 1{\cdot}366 \left(\frac{\alpha}{\alpha'}\right)^{0{\cdot}549}; \quad \alpha' = 0{\cdot}00305.$$

In this equation m and B_m represent the observed limiting magnitudes and star counts (number of stars in 1 square degree), and $\bar{M}_0$, α are quantities depending on the galactic co-ordinates.

Now, Professor Turner, in Table I. of the paper already quoted, has given the numbers referring to the adopted steps A, B, etc., which were counted in areas of 40 square degrees centred at R.A. 0^h, 1^h . . . 23^h. Accepting his magnitudes [(C. & M.) corrected to the Mount Wilson scale], which he derived from the general material (see last horizontal column of Table I.), each horizontal row gives a relation between B_m and m for the areas 0^h, 1^h, etc., from which $\bar{M}_0$, α may be calculated. To smooth out accidental errors, three adjacent areas were combined, and thus the following results were obtained :—

R.A. h	Galactic Latitude, β.	$\bar{M}_0$	α.
1	76	4·25	0·00524
4	41	4·54	0·00380
7	9	3·89	0·00204
10	21	3·81	0·00303
13	28	3·93	0·00347
16	9	3·58	0·00246
19	24	4·12	0·00221
22	61	4·61	0·00390

Computing, by means of the derived quantities $\bar{M}_0$, α and the observed values of B_m, the magnitudes m and comparing with the observed limiting magnitudes, the following residuals $(o - c)$ were obtained :—

	h 1.	h 4.	h 7.	h 10.	h 13.	h 16.	h 19.	h 22.	Mean.
Br.	+0·03	+0·14	+0·01	+0·03	−0·11	−0·06	+0·09	−0·01	+0·01
A	+0·16	−0·04	−0·04	0·00	0·00	−0·03	−0·09	+0·14	+0·01
B	−0·14	−0·02	+0·04	−0·03	+0·07	+0·03	−0·09	+0·03	−0·01
C	−0·05	−0·01	+0·02	+0·02	+0·06	+0·01	−0·02	−0·06	0·00
D	−0·08	−0·05	+0·02	−0·01	+0·09	0·00	+0·04	−0·07	−0·01
E	−0·09	−0·07	0·00	+0·02	+0·03	+0·02	+0·04	−0·14	−0·02
F	−0·10	−0·08	−0·10	0·00	0·00	+0·03	+0·07	−0·08	−0·03
G	+0·02	+0·02	−0·02	−0·03	−0·05	−0·02	+0·07	+0·01	0·00
H	+0·04	+0·07	+0·01	+0·02	−0·02	−0·04	+0·02	+0·06	+0·02
J	+0·07	+0·06	+0·03	+0·05	+0·03	−0·01	−0·10	+0·10	+0·03

It was shown in the author's paper already referred to that the quantities α derived from Chapman and Melotte's figures are well represented by the relation

$$\alpha = +0·00183 + 0·00307 \sin \beta,$$

where β is the galactic latitude.

Analysing the quantities a obtained from the material of the Perth zone with regard to β, we get

$$a = +0{\cdot}00168 + 0{\cdot}00318 \sin \beta,$$

in sufficient agreement with Chapman and Melotte.

The changes of $\bar{M}_0$ with galactic latitude are also in agreement with those derived from the Chapman and Melotte figures, but somewhat more pronounced. The following table gives a comparison between the two results:—

Chapman and Melotte.			Perth Zone.		
β.	$\bar{M}_0$.	a.	β.	$\bar{M}_0$.	a.
5°	3·82	0·00228	9°	3·79	0·00225
15	3·86	0·00254	22	3·96	0·00262
25	3·87	0·00296	35	4·23	0·00364
35	3·98	0·00356	69	4·43	0·00457
45	4·09	0·00429			
55	4·20	0·00402			
65	4·22	0·00435			
80	4·17	0·00487			

In view of this agreement, the general conclusion appears warranted that the results derived from the Perth zone confirm essentially and in every respect the distribution shown in Chapman and Melotte's tables down to the 13th magnitude.

This conclusion is in accordance with results derived from a discussion of the distribution of the stars in the C.P.D. (see the author's paper: "On the Magnitudes of the Cape Photographic Durchmusterung").

The following reports of the Sydney and Adelaide Observatories were received too late for insertion in the Annual Report of the Council:—

Sydney Observatory.

(*Director, Mr. W. Ernest Cooke, Government Astronomer.*)

The energies of the staff have been mainly concentrated upon the three sections of accumulated astrographic work with the following results:—

(1) Great progress has been made with the meridian reductions. They are now sufficiently in hand to permit a start to be made very shortly upon a fresh observational programme.

(2) The plate measures are being prepared for publication, and 536 regions are now ready for the printer.

(3) Pairs of chart plates separated by an interval of ten years or more have been selected and sent to Mr. Innes of the Union

Observatory, South Africa, who has kindly consented to observe the complete available series with the blink microscope. Already 1½ zones, containing 150 regions, have been completely examined.

During the latter part of 1916 the project for removing the Observatory to a new site and equipping it with modern instruments was definitely abandoned for the present, and arrangements commenced for the astronomer to occupy the old quarters at the Observatory, which had for several years been used as offices by the Meteorological Department. These arrangements were not completed until about 1917 September, but the astronomer is at last, after five years' drifting about, living at the Observatory.

During the year preparations were made for the new meridian programme, which will consist of observations of the intermediate stars between 50° and 65° south declination. These include various alterations to the instrument, and the computation of tables, etc., for the new methods which are to be adopted. Preliminary observations have been taken, and a definite start is expected early in 1918. During the first year the pace will not be very rapid, as the observers are also computers, and progress with the back accumulations must be maintained.

The astrograph is also practically completed for a fresh start, but this is still being delayed by circumstances beyond the astronomer's control.

No definite programme has yet been arranged for the 11½-inch equatorial, for two reasons:

(1) The dome is almost unworkable.

(2) The astrographic work, both past and future, has first claim; and until the new observational programme is in full swing, it is unwise to fritter away our limited energy upon anything else. The keynote of the Observatory at present is concentration upon the astrographic work, including of course the meridian portion.

The usual routine duties, including University lectures, time-service, seismograph, reception of visitors, etc., have been maintained as heretofore.

Adelaide Observatory, South Australia.
(*Director, Mr. G. F. Dodwell, Government Astronomer.*)

Meridian and Time-service.—The transit-circle telescope was used for observation of clock stars and intermediate standard stars of the Sydney Zone ($-52°$ to $-64°$). There was a break in the continuity of the latter, however, owing to the enlistment of the senior assistant observer (Mr. A. E. Messent) in the Australian Imperial Forces. Later in the year the staff was augmented by the appointment of student evening observers from the Adelaide University to assist in the evening observations, and the zone work was taken up again. Professor Cooke's differential tables for R.A. reductions were found very useful, and were supplemented during the year by similar tables for N.P.D.'s.

The time-ball at the semaphore was dropped at 1 p.m. daily (S.A. clock time), Sundays excepted. From January 1st to March 25th, while the Daylight Saving Act was in force, this corresponded to 14 hours 30 minutes G.M.T., and after the latter date to 15 hours 30 minutes G.M.T. As the Daylight Saving Act has been repealed, no alteration has been made this summer.

On occasions when telegraphic communication was temporarily suspended the semaphore time-ball was dropped by hand by the signalman on a signal given through the telephone from the Observatory. The telephone and push-button used in this connection have been installed close to the releasing-trigger. The noise made by the mechanism when the ball is released can be heard at the Observatory through the telephone, and on each occasion when this method has been used an indication has been obtained that the ball has been correctly dropped. The return-signal apparatus has worked well, and the value previously adopted for internal lag of the mechanism (0·3 second) has been confirmed. This amount has been allowed for each day. The transmission of time-signals to the Adelaide Radio station and thence by wireless to shipping has been continued, the signals being those adopted by the International Time Convention. They were emitted throughout the year at midday and midnight, Adelaide standard time, corresponding to 14 hours 30 minutes G.M.T. and 2 hours 30 minutes G.M.T. respectively, no alteration being made during the "Daylight Saving" period. On November 8th a time-ball was established for shipping at Port Pirie. The ordinary system of distributing the time to country telegraph stations and various Government departments was continued.

Field Work.—Latitude, longitude, and magnetic stations were established at Mount Gambier, Penola, Robe, Long Gully, S.E., Kingston, Kybybolite, Murray Bridge, Blanchetown, and Port Pirie. For the magnetic work a magnetometer outfit, lent by the Carnegie Institution, and dip circle, lent by the Sydney Observatory, were used. Professor Kerr Grant of the Adelaide University co-operated in this work at all but one of the above stations, which were occupied during the University vacations. Mr. F. B. Cooke of the Sydney Observatory assisted in the work at Penola in addition to making a personal equation determination in the field and at the Adelaide Observatory, to connect up longitude work done at Sydney and Adelaide. For latitude and longitude the almucantar method of observing was used, and longitude signals from the Adelaide Observatory *via* the Radio station were received by means of a portable wireless receiving outfit. Clock signals from Sydney Observatory *via* Pennant Hills Radio station were received at Penola. A permanent mark at the site of the observations and concrete block to indicate the meridian have been fixed at the latitude and longitude stations.

Seismology.—The Milne seismograph was continually at work during the year, and the tabulated records were sent as usual to the Shide Observatory, Isle of Wight.

Miscellaneous.—The Board of Visitors held their annual meeting at the Observatory on November 23rd.

Mr. C. A. Maddern, a member of the staff, has been at the front with the Australian Imperial Forces since 1915 December. Mr. A. E. Messent, senior assistant observer, enlisted, and was absent on military service from March to November.

The wireless receiving installation at the Observatory was made more efficient by erecting a new mast 73 feet 6 inches high for the aerial, which was previously attached to the wind tower. An umbrella-type aerial, designed by Mr. F. B. Cooke, was at the same time introduced in place of the L aerial formerly used.

The equatorial telescope has been used principally in connection with the regular Friday night visits by the public, and also on certain evenings by members of the S.A. Astronomical Society for variable and double star observations.